ESISAR 5A
Department : AUTOMATIQUE
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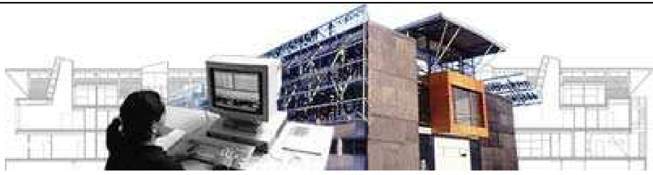
2023 : Fault Diagnosis



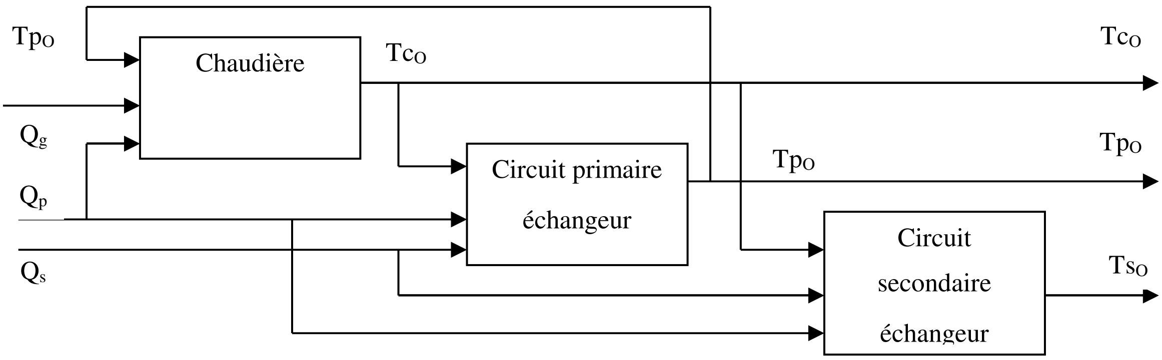
TP 3: PARITY SPACE APPROACH

Objective: Build a bank of residuals in order to detect and isolate the fault sensor

1 SENSOR FAULT DETECTION AND ISOLATION FOR A BOILER SYSTEM EXCHANGER: PARITY SPACE APPROACH

Theoretical study

Consider the following boiler system exchanger



The variables are:

- T_c : output water temperature at the boiler °C
- T_p : output water temperature of the primary circuit of the exchanger °C
- Ts : output water temperature of the second circuit of the exchanger °C
- Q_g : gas flow m^3/h
- Q_p : water flow of the primary circuit of the exchanger l/h
- Q_s: water flow of the second circuit of the exchanger l/h

The state of the system is desbribed by the following relation :

$$x(k) = \left(T_c(k)\ T_p(k)\ T_s(k)\right)^T$$

the control inputs by :

$$u(k) = \left(Q_g(k-2)\ Q_p(k-1)\ Q_p(k-5)\ Q_s(k-1)\right)$$

And the output by :

$$y(k) = \left(T_c(k) \quad T_p(k) \quad T_s(k)\right)^T$$

The matrices A, B and C are given by :

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{bmatrix}, \; B = \begin{bmatrix} b_{11} & 0 & b_{13} & 0 \\ 0 & b_{22} & 0 & b_{24} \\ 0 & b_{32} & 0 & b_{34} \end{bmatrix}, \; C = I_3$$

Using a identification process, web obtain the matrices A and B

$$A = \begin{bmatrix} 0.9494 & 0.0718 & 0 \\ 0.2363 & 0.5801 & 0 \\ 0.2388 & 0 & 0.5483 \end{bmatrix} \; B = \begin{bmatrix} 0.0557 & 0 & -0.0068 & 0 \\ 0 & 0.0329 & 0 & -0.0112 \\ 0 & 0.0132 & 0 & -0.0429 \end{bmatrix}$$

1.1 Parity space

- Without increasing the time k, how many parity equations can be deduced?
- Gives the parity equations with a minimum horizon of observation, to limit the detection delays.
- Gives the fault location decision table, each sensor fault alarm with the logic decision.
- Using matab/simuling gives the implementation solution

2 STATIC PARITY SPACE WITH NON PERFECT DECOUPLING

Consider the static output relation:

$$y(k) = Cx(k) + \varepsilon(k) + Fd(k)$$

where $x \in R^n$, $y \in R^m$ and $d \in R^p$.

y(k) is the output measurement, x(k) the state variable, d(k) the fault vector to be detected and ε(k) the noise measurement. Matrices C and F are known with appropriate dimension.

Assumption: m > n for a redundancy information existence.

We will find a parity vector sensitive to p-1 defaults and insensitive to d_i which represent the ith component of d.

This leads to explain the measurement vector in the form:

$$y(k) = Cx(k) + \varepsilon(k) + F^+d^+(k) + F^-d^-(k)$$

where d^+ are d^- are respectively the sensitive and insensitive faults. F^+ et F^- are the associated matrices.

2.1 Numerical Application:

Consider the following system:

$$C = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 0 & 2 \end{pmatrix}, \quad F^- = \begin{pmatrix} 1 & 2 \\ 1 & 2 \\ 0 & 0 \\ 2 & 5 \\ 0 & 1 \end{pmatrix}, \quad F^+ = \begin{pmatrix} 1 \\ 0 \\ 3 \\ 1 \\ 1 \end{pmatrix}$$

- Find the kernel w with the optimization problem described bellow. Give w^TF^- and w^TF^+.

- Check the products w^TC, w^TF^- and w^TF^+ and conclude.

Solution: The problem is reduced to find a matrices W such that

$$W\begin{pmatrix}C & F^-\end{pmatrix} = 0 \qquad (1)$$

where the parity vector is given by : $p(k) = W\varepsilon(k) + WF^+d^+(k)$

The exact solution W of (1) holds if and if the line rank of (C F^-) is deficient. If not the following optimization problem can be used:

$$\begin{cases} w^TC = 0 \\ \min\limits_{w} \dfrac{\left\|w^TF^-\right\|^2}{\left\|w^TF^+\right\|^2} \end{cases}$$

remark: max(f(x)) = - min(-f(x)) example f(x) = x^2-2

Explanation

Consider $C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$ where C1 is of full rank. The constraints $w^TC = 0$ is true by using the following changement :

$$w = Pw_2$$

where $P = \begin{pmatrix} -\left(C_1^{-1}\right)^T C_2^T \\ I \end{pmatrix}$.

<u>Proof</u> : $w^TC = 0 \Leftrightarrow \begin{pmatrix} w_1^T & w_2^T \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = 0 \Leftrightarrow w_1^T = -w_2^T C_2 C_1^{-1}$ thus $w_2^T\begin{pmatrix} -C_2C_1^{-1} & I \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = 0$.

With identification: $w^T = w_2^T\begin{pmatrix} -C_2C_1^{-1} & I \end{pmatrix} \Leftrightarrow w = \begin{pmatrix} -\left(C_1^{-1}\right)^T C_2^T \\ I \end{pmatrix} w_2 \Leftrightarrow w = Pw_2$.

Using the new variable w_2, the problem $w^TC = 0$; $\min\limits_{w} \dfrac{\left\|w^TF^-\right\|^2}{\left\|w^TF^+\right\|^2}$ becomes :

$$\min_{w} \frac{\left\|w^TF^-\right\|^2}{\left\|w^TF^+\right\|^2} = \min_{w} \frac{w^TF^-\left(F^-\right)^T w}{w^TF^+\left(F^+\right)^T w} = \min_{w_2} \frac{w_2^TP^TF^-\left(F^-\right)^T Pw_2}{w_2^TP^TF^+\left(F^+\right)^T Pw_2} = \min_{w_2} \frac{w_2^T\tilde{A}w_2}{w_2^T\tilde{B}w_2}$$

where $\tilde{A} = P^TF^-\left(F^-\right)^T P$ and $\tilde{B} = P^TF^+\left(F^+\right)^T P$.

Since the small eigenvalue λ of the pair ($\tilde{A}, \tilde{B}$) is the minimal of the criteria, the corresponding eigenvector w_2 is the solution of the optimization problem. Find λ, such that $\left(\tilde{A} - \lambda\tilde{B}\right)w_2 = 0$. From w_2 and P deduce w and the parity relation:

$$\begin{aligned} p &= w^T\underline{y} \\ p &= w^T\begin{pmatrix} F^- & F^+ \end{pmatrix}\begin{pmatrix} d^- \\ d^+ \end{pmatrix} \end{aligned}$$

That conclude the proof.