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Brief paper

Filtering and fault estimation of descriptor switched systems ☆

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ABSTRACT

In this paper, the problems of state and fault estimation are addressed for a class of switched descriptor systems subject to Lipschitz nonlinearities and unknown inputs (UI). The UI appear both on the dynamic and on the measurement equations. Two problems are addressed by $\mathcal{L}_2$-gain minimization with the use of switched Lyapunov functions and formulated by LMI. First, a functional observer for switched Lipschitz nonlinear descriptor system is proposed for robust state estimation. Second, fault estimation is performed by filtering the output estimation error, as usually done in the residual generation framework. Moreover, frequency weighting functions can be used to shape the response to the fault and thus improve the estimation.

1. Introduction

Controller and/or observer design for switched systems has recently received much attention. Switched systems are a class of hybrid systems defined by a collection of dynamical (linear and/or nonlinear) subsystems together with a switching rule specifying the switching between the subsystems. Surveys on switched systems are available in Cortes (2008), Lieberzon and Morse (1999) and Sun and Ge (2005). Many arising problems were treated for switched systems: stability (Johansson & Rantzer, 1998; Sun, Wang, & Xie, 2006), output-feedback (Daafouz, Riedinger, & Iung, 2002; Farral, Mhaskar, & Christofides, 2005), state estimation (Koenig, Marx, & Jacquet, 2008).

The descriptor systems generalize the state-space systems by encompassing both differential and static relations (Dai, 1989; Luenberger, 1977; Xu & Lam, 2006). Many results have been extended to descriptor systems concerning stability, control or state estimation (Dai, 1989; Masubuchi, Kamitane, Ohara, & Suda, 1997), fault estimation (Koenig, 2005) and fault tolerant control (Gao & Ding, 2007; Marx, Koenig, & Georges, 2004).

The motivation of the present work is to extend some results on state and fault estimation to switched descriptor systems subjected to unknown inputs (UI), faults and Lipschitz nonlinearities. This formalism allows to model systems with both dynamic and static behaviors, with functioning mode changes and which inputs are partially unavailable to measurement (fault, disturbance, etc.). Despite its generality, only few results exist for the class of discrete time switched descriptor systems (DTSDS). In Haidar and Boukas (2008) and Ma and Boukas (2008) stability of Markovian descriptor systems are studied. In Koenig and Marx (2009), H_∞-filtering and state feedback are treated but no nonlinearities are considered. Proposing a unified approach to robust state filtering and fault diagnosis for DTSDS, two objectives are aimed here. The first objective is to relax the perfect UI decoupling conditions needed for state estimation in Koenig et al. (2008) by using the $\mathcal{L}_2$-approach. Moreover, in order to obtain relaxed stability conditions, multiple Lyapunov functions are used to derive LMI conditions and introduce some non necessary positive definite slack variables. The second one is to perform robust fault diagnosis via robust fault estimation and to generalize (Koenig et al., 2008), where only disturbance UI were envisaged, but no diagnosis was performed. The objective of fault diagnosis is to highlight the faults (actuator or sensor dysfunction) while being robust to the disturbance UI (Blanke, Kinnaert, Lunze, & Staroswiecki, 2006). In observer-based fault diagnosis, the output estimation error is usually used as a primary residual signal. This primary residual signal, affected by both disturbance and fault, is filtered by a post-filter to obtain a robust fault estimate. The state and fault estimation are performed in a unified way since

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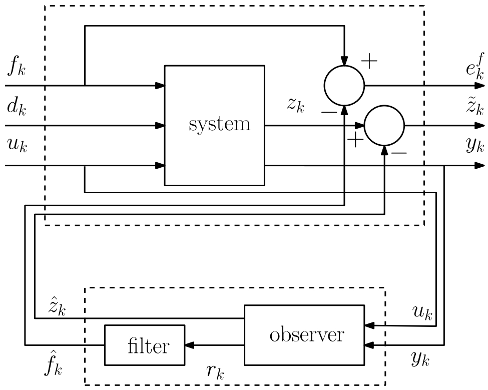


Fig. 1. State filtering and fault estimation scheme.

the generating systems of their respective error estimations is written similarly. One should note that no previous works have considered the problem of robust fault diagnosis for nonlinear switched descriptor systems.

This note is organized as follows. The problem is stated in Section 2. In Section 3, the state and fault observers are designed. Before concluding, Section 4 is devoted to a numerical example.

Notation 1. *For any square matrix M, $M > 0$ (resp. $M < 0$) means that the matrix M is a real symmetric positive (resp. negative) definite and $\mathbb{S}(M)$ is defined by $\mathbb{S}(M) = M + M^T$. The blocks induced by symmetry are denoted $*$, I_n is the $n \times n$ identity matrix, 0_n (resp. $0_{n\times m}$) is the $n \times n$ (resp. $n \times m$) null matrix and $\mathrm{diag}(X_1, \ldots, X_n)$ is the block diagonal matrix which diagonal entries are $X_1, \ldots, X_n$. The set of the N first strictly positive integers is denoted $\mathbf{N}_N = \{1, \ldots, N\}$ and $\ell_2\,[0, \infty)$ denotes the space of square summable infinite vector sequences with the usual norm $\|.\|_2$.*

2. Problem formulation

Consider the following DTSDS:

$$\sum_{j=1}^{N} \alpha_j(k+1) E_j x_{k+1} = \sum_{i=1}^{N} \alpha_i(k) \big(A_i x_k + B_i u_k + R_i^1 f_k + W_i^1 d_k + H_i \phi(x_k, u_k, k)\big) \tag{1a}$$

$$y_k = \sum_{i=1}^{N} \alpha_i(k) \left(C_i x_k + D_i u_k + R_i^2 f_k + W_i^2 d_k\right) \tag{1b}$$

$$z_k = \sum_{i=1}^{N} \alpha_i(k) T_i x_k \tag{1c}$$

where $x \in \mathbb{R}^n$, $d \in \mathbb{R}^{n_d}$, $f \in \mathbb{R}^{n_f}$, $u \in \mathbb{R}^{n_u}$, $\phi : \mathbb{R}^n \times \mathbb{R}^{n_u} \times \mathbb{N} \to R^{n_\phi}$, $y \in \mathbb{R}^m$ and $z \in \mathbb{R}^q$ denote respectively the state, the UI, the fault, the control input, the Lipschitz nonlinearity, the output vector and the vector to be estimated, with $q \leq n$. The matrices E_j may be singular. The functions $\alpha_i : \mathbb{N} \to \{0, 1\}$ $(i \in \mathbf{N}_N)$ are the known switching signals satisfying $\sum_{i=1}^{N} \alpha_i(k) = 1$ $(k \in \mathbb{N})$ and specifying the activated subsystem: $\alpha_i(k) = 1$ and $\alpha_j(k+1) = 1$ mean that the matrices $(E_j, A_i, B_i, W_i, C_i)$ are activated at time k. The measurements y, the vector z, the fault f and the UI d are respectively assumed to be linearly independent *i.e.*

$$\mathrm{rank}\left(\begin{bmatrix} C_i & D_i & R_i^2 & W_i^2 \end{bmatrix}\right) = m$$
$$\mathrm{rank}\,(T_i) = q$$
$$\mathrm{rank}\left(\begin{bmatrix} R_i^1 & W_i^1 \\ R_i^2 & W_i^2 \end{bmatrix}\right) = n_f + n_d.$$

Assumptions. In the sequel it is assumed that:

(A1) the nonlinearity $\phi(x_k, u_k, k)$ is globally Lipschitz in x_k *i.e.* there exist a constant β s.t. $\forall u_k \in \mathbb{R}^{n_u}$ and $\forall k \in \mathbb{N}$

$$\left\|\phi(x_k, u_k, k) - \phi(\hat{x}_k, u_k, k)\right\| \leq \beta \left\|x_k - \hat{x}_k\right\| \tag{2}$$

(A2) for $i \in \mathbf{N}_N$, the triplet (E_i, A_i, C_i) is impulse observable and detectable (Dai, 1989).

Problem 1. Consider the switched functional observer (SFO) for the fault free DTSDS (1)

$$\sum_{j=1}^{N} \alpha_j(k+1) E_j \hat{x}_{k+1} = \sum_{i=1}^{N} \alpha_i(k) \Big(A_i \hat{x}_k + B_i u_k + H_i \phi(\hat{x}_k, u_k, k) + L_i r_k\Big) \tag{3a}$$

$$\hat{y}_k = \sum_{i=1}^{N} \alpha_i(k) \left(C_i \hat{x}_k + D_i u_k\right) \tag{3b}$$

$$\hat{z}_k = \sum_{i=1}^{N} \alpha_i(k) T_i \hat{x}_k \tag{3c}$$

$$r_k = y_k - \hat{y}_k \tag{3d}$$

where L_i are the observer gains, $\hat{x}$ and $\hat{z}$ are the estimate of x and z and r is the output estimation error. The gains L_i are determined such that:

(S_1) the state estimation error, $e_k = x_k - \hat{x}_k$, is generated by a globally asymptotically stable and impulse free system, when ${d_k}^T = 0$ and ${f_k}^T = 0$;

(S_2) the $\mathcal{L}_2$-gain from the UI d_k to the estimation error $\tilde{z}_k = z_k - \hat{z}_k$ is bounded by a prescribed positive scalar γ_1.

Problem 2. A post-filter is designed in order to estimate the fault. The proposed SFO and post-filter are defined by (3) and (4) respectively.

$$x_{k+1}^F = \sum_{i=1}^{N} \alpha_i(k) \left(A_i^F x_k^F + B_i^F r_k\right) \tag{4a}$$

$$\hat{f}_k = \sum_{i=1}^{N} \alpha_i(k) \left(C_i^F x_k^F + D_i^F r_k\right) \tag{4b}$$

where $\hat{f}_k$ is the fault estimate and $x_k^F \in \mathbb{R}^{n_F}$ is the filter state. The problem is then to simultaneously determine the gains L_i and the matrices A_i^F, B_i^F, C_i^F and D_i^F satisfying the following specifications.

(S_3) the state and fault estimation errors ($e_k = x_k - \hat{x}_k$ and $e_k^f = f_k - \hat{f}_k$) are generated by a globally asymptotically stable and impulse free system, when $d_k = 0$ and $f_k = 0$;

(S_4) the $\mathcal{L}_2$-gain from $(w_k)^T = \left[(d_k)^T \ (f_k)^T\right]$ to the fault estimation error e_k^f is bounded by a prescribed positive scalar γ_2.

This design procedure can be viewed as a standard H_∞-control problem, as shown in Fig. 1.

3. Robust functional observer design and fault diagnosis

The gains of the SFO (3) and filter (4) for the DTSDS (1) are obtained by solving an LMI problem. Two Lyapunov candidate functions are considered:

$$V(e_k) = \sum_{i=1}^{N} \alpha_i(k) {e_k}^T {E_i}^T P_i E_i e_k \tag{5}$$

$$V^a(e_k^a) = \sum_{i=1}^{N} \alpha_i(k) e_k^{aT} E_i^{aT} P_{a_i} E_i^a e_k^a \tag{6}$$

where P_i and P_{a_i} are symmetric matrices and where $(e_k^a)^T = \left[(e_k)^T\ (x_k^F)^T\ (e_k^f)^T\right]$. It is well-known (Boyd, Ghaoui, Feron, & Balakrishnan, 1994; Chadli & Darouach, 2011) that for such Lyapunov functions:

- $\Delta V(e_k) = V(e_{k+1}) - V(e_k) < 0$ implies (S_1)
- $H_{e_k}(\gamma_1) = \Delta V(e_k) + \tilde{z}_k^T\tilde{z}_k - {\gamma_1}^2 {d_k}^T d_k < 0$ implies (S_2)
- $\Delta V^a(e_k^a) = V^a(e_{k+1}^a) - V^a(e_k^a) < 0$ implies (S_3)
- $H_{e_k^a}(\gamma_2) = \Delta V^a(e_k^a) + {e_k^f}^T e_k^f - \gamma_2^2 {w_k}^T w_k < 0$ implies (S_4).

3.1. State filtering in the fault free case

The first objective is to develop a SFO (3) for the fault free DTSDS (1), such that the specifications (S_1) and (S_2) are fulfilled. The filtering error between (3) and (1) is generated by

$$\sum_{j=1}^{N} \alpha_j(k+1) E_j e_{k+1} = \sum_{i=1}^{N} \alpha_i(k) \left(A_{cl_i} e_k + H_i \tilde{\phi}_k + W_{cl_i} d_k\right) \tag{7a}$$

$$\tilde{z}_k = \sum_{i=1}^{N} \alpha_i(k) T_i e_k \tag{7b}$$

where $\tilde{\phi}_k = \phi(x_k, u_k, k) - \phi(\hat{x}_k, u_k, k)$, $A_{cl_i} = (A_i - L_i C_i)$ and $W_{cl_i} = (W_i^1 - L_i W_i^2)$.

The following result details sufficient LMI existence conditions of the SFO and the computation of the gains L_i.

Theorem 1. *The SFO (3) for the DTSDS (1), satisfying $(S1)$ and $(S2)$ exists if the triplets (E_i, A_i, C_i) are finite dynamics detectable and impulse observable (see Koenig & Marx, 2009 for conditions) and is obtained by finding the symmetric matrices $P_i \in \mathbb{R}^{n\times n}$, symmetric positive definite matrices $G_i \in \mathbb{R}^{n\times n}$, matrices $\tilde{G}_i \in \mathbb{R}^{n\times m}$ and $M_i \in \mathbb{R}^{(n+n_d+n_\phi)\times n}$ minimizing $\bar{\gamma}_1$ under the constraints (8a)–(8c) for $(i,j) \in \mathbf{N}_N^2$.*

$$E_i^T P_i E_i \geq 0 \tag{8a}$$

$$\mathcal{M}_{iji} < 0 \tag{8b}$$

$$\mathcal{M}_{ijj} < 0 \tag{8c}$$

where $\bar{\gamma}_1 = \gamma_1^2$ and

$$\mathcal{M}_{ijk} = \begin{bmatrix} \Theta_{ik} & \bar{\Theta}_i & M_i & \tilde{C}_i^T \tilde{G}_i^T \\ * & P_j - 2G_i & 0 & 0 \\ * & * & -G_i & 0 \\ * & * & * & -G_i \end{bmatrix}$$

$$\bar{\Theta}_i = (\tilde{A}_i^T G_i^T - \tilde{C}_i^T \tilde{G}_i^T - M_i)$$

$$\Theta_{ik} = \tilde{T}_{ik} + \tilde{C}_i^T C_i G_i {C_i}^T \tilde{C}_i + \mathbb{S}\left(M_i \tilde{A}_i + M_i {C_i}^T \tilde{C}_i + \tilde{C}_i^T C_i \tilde{G}_i \tilde{C}_i\right)$$

$$\tilde{T}_{ik} = \mathrm{diag}\left(\beta^2 I_n + {T_i}^T T_i - {E_k}^T P_i E_k, -\bar{\gamma}_1 I_{n_d}, -I_{n_\phi}\right)$$

$$\tilde{A}_i = \begin{bmatrix} A_i & W_i^1 & H_i \end{bmatrix}$$

$$\tilde{C}_i = \begin{bmatrix} C_i & W_i^2 & 0 \end{bmatrix}.$$

The observer gains are obtained by

$$L_i = G_i^{-1} \tilde{G}_i. \tag{9}$$

Proof. The disturbance attenuation of the DTSDS (7) expressed by S_2 has to be satisfied under arbitrary switching laws, it follows that $H_{e_k}(\gamma_1) < 0$ and (7) are respectively equivalent to

$$e_{k+1}{}^T E_j{}^T P_j E_j e_{k+1} - {\gamma_1}^2 {d_k}^T d_k - {e_k}^T \left({E_i}^T P_i E_i - {T_i}^T T_i\right) e_k < 0 \tag{10}$$

and

$$E_j e_{k+1} = A_{cl_i} e_k + H_i \tilde{\phi}_k + W_{cl_i} d_k \tag{11a}$$

$$\tilde{z}_k = T_i e_k \tag{11b}$$

where i (resp. j) is the number of the activated model at time k (resp. $k+1$). Substituting (11) into (10), the following inequality is obtained:

$$H_{e_k}(\gamma_1) = (*) P_j \left[A_{cl_i} e_k + H_i \tilde{\phi}_k + W_{cl_i} d_k\right] + {e_k}^T \left({T_i}^T T_i - {E_i}^T P_i E_i\right) e_k - {\gamma_1}^2 {d_k}^T d_k < 0. \tag{12}$$

Defining $\tilde{\Theta}_{ij} = \Theta_{ij} + M_i G_i^{-1} {M_i}^T + \tilde{C}_i^T \tilde{G}_i^T G_i^{-1} \tilde{G}_i \tilde{C}_i$, if (8b) hold and with two Schur complements, (8b) is equivalent to

$$\begin{bmatrix} \tilde{\Theta}_{ii} & \tilde{A}_i^T {G_i}^T - \tilde{C}_i^T \tilde{G}_i^T - M_i \\ * & P_j - 2G_i \end{bmatrix} < 0. \tag{13}$$

From (8b), the matrices $G_i > 0$ and defining $\tilde{M}_i = {M_i}^T + \tilde{G}_i \tilde{C}_i + G_i {C_i}^T \tilde{C}_i$, it follows $\tilde{M}_i^T G_i^{-1} \tilde{M}_i \geq 0$ or equivalently

$$-\mathbb{S}\left(M_i G_i^{-1} \tilde{G}_i \tilde{C}_i\right) \leq M_i G_i^{-1} {M_i}^T + \tilde{C}_i^T \tilde{G}_i^T G_i^{-1} \tilde{G}_i \tilde{C}_i + \tilde{C}_i^T C_i G_i {C_i}^T \tilde{C}_i + \mathbb{S}\left(M_i {C_i}^T \tilde{C}_i + \tilde{C}_i^T C_i \tilde{G}_i \tilde{C}_i\right). \tag{14}$$

Adding $\tilde{T}_{ii} + \mathbb{S}(M_i \tilde{A}_i)$ to both sides of (14), it follows

$$\tilde{T}_{ii} + \mathbb{S}(M_i \tilde{A}_i - M_i G_i^{-1} \tilde{G}_i \tilde{C}_i) \leq \tilde{\Theta}_{ij} < 0. \tag{15}$$

From (15), with (9) and $\tilde{A}_{cli} = \tilde{A}_i - L_i \tilde{C}_i$, it follows that (13) implies

$$\begin{bmatrix} \tilde{T}_{ii} + \mathbb{S}(M_i \tilde{A}_{cli}) & \tilde{A}_{cli}^T {G_i}^T - M_i \\ * & P_j - 2G_i \end{bmatrix} < 0. \tag{16}$$

Pre- and post-multiplying (16) by $\left[I\ \tilde{A}_{cli}^T\right]$ and its transpose, it follows that (16) is equivalent to

$$\tilde{T}_{ii} + \tilde{A}_{cli}^T P_j \tilde{A}_{cli} < 0. \tag{17}$$

Pre- and post-multiplying (17) by $\left[{e_k}^T\ {d_k}^T\ \tilde{\phi}_k^T\right]$ and its transpose, (17) becomes

$$H_{e_k}(\gamma_1) + \beta^2 {e_k}^T e_k - \tilde{\phi}_k^T \tilde{\phi}_k < 0 \tag{18}$$

where $H_{e_k}(\gamma_1)$ is defined by (12). Since the nonlinearity is assumed to be Lipschitz in x (2), then

$$\beta^2 {e_k}^T e_k - \tilde{\phi}_k^T \tilde{\phi}_k \geq 0. \tag{19}$$

From (18) and (19) it follows that $H_{e_k}(\gamma_1) < 0$ and thus (S_2) is satisfied. Moreover, when $d_k = 0$ in $H_{e_k}(\gamma_1)$, it obviously follows that $\Delta V(e_k) < 0$. From Definition 1.1 of Chadli and Darouach (2011) the system (11) is stable. Following the same steps, (8c) implies $\tilde{T}_{ij} + \tilde{A}_{cli}^T P_j \tilde{A}_{cli} < 0$. Pre- and post multiplying this inequality by $[I_n\ 0_{n_d}\ 0_{n_\phi}]$ and its transpose, one obtains $(A_i - L_i C_i)^T P_j (A_i - L_i C_i) - E_j^T P_i E_j < 0$ for $(i,j) \in \mathbf{N}_N^2$, implying that (11) is impulse free. Thus (S_1) holds which achieves the proof. □

Remark 1. The objective is the minimization of the $\mathcal{L}_2$-gain from d_k to e_k, consequently the perfect decoupling is not sought and the conditions assumed in Koenig et al. (2008) are not needed to solve the LMIs (8).

The special case when the system is neither affected by disturbances nor by nonlinearities (i.e. $f_k = 0$, $d_k = 0$ and $\phi(x_k, u_k, k) = 0$) is briefly envisaged in the following corollary.

Corollary 1. *A SFO (3) for the DTSDS (1) with* $f_k = d_k = \phi(x_k, u_k, k) = 0$ *exists and satisfies* (S_1) *if there exist symmetric matrices* $P_i \in \mathbb{R}^{n\times n}$, *matrices* $U_i \in \mathbb{R}^{n\times n}$ *and* $M_i \in \mathbb{R}^{m\times n}$ *satisfying (8) for* $(i,j) \in \mathbf{N}_N^2$ *with* $\mathcal{M}_{ijk}$ *defined by*

$$\mathcal{M}_{ijk} = \begin{bmatrix} \bar{\Theta}_{ik} & * \\ -U_i^T + U_iA_i - M_iC_i & P_j - \mathbb{S}(U_i) \end{bmatrix} \tag{20a}$$

$$\bar{\Theta}_{ik} = \mathbb{S}(U_iA_i - M_iC_i) - E_k^T P_i E_k. \tag{20b}$$

The observer gains are: $H_i = 0$ *and* $L_i = U_i^{-1}M_i$.

Proof. Consider (8b) defined with (20) and $U_iL_i = M_i$, pre- and post-multiplying it by $\left[I_n \; A_{cli}{}^T\right]$ and its transpose, then $A_{cli}{}^T P_j A_{cli} - E_i{}^T P_i E_i < 0$ and consequently $\Delta V(e_k) < 0$ follows. From (8c), impulse freeness is obtained like in the proof of Theorem 1 and (S_1) follows. □

Remark 2. For $A_{cl_i} = A$, $E_i = E$ and $P_i = P_j = P$, the inequalities (8) with (20) imply the LMIs defined in Lemma 1 of Xu and Yang (1999). For $A_{cl_i} = A_i$ and $E_i = E$, they are equivalent to the LMIs defined in Xu and Lam (2006). For $E_i = E$ and by the duality principle, the results of Corollary 1 coincides with the results of Theorem 1 of Chadli, Daafouz, and Darouach (2008). Thus, Corollary 1 can be considered as a generalization of these works.

3.2. Fault estimation

In order to simultaneously design the SFO (3) and the filter (4), the DTSDS generating the state and fault estimation errors is written as

$$\sum_{j=1}^{N} \alpha_j(k+1)E_j^a e_{k+1}^a = \sum_{i=1}^{N} \alpha_i(k)\left(A_{cli}^a e_k^a + W_{cli}^a w_k + H_i^a \tilde{\phi}_k\right) \tag{21a}$$

$$e_k^f = \sum_{i=1}^{N} \alpha_i(k) T_i^a e_k^a \tag{21b}$$

where $e_k^{aT} = \left[e_k{}^T \; x_k^{F^T} \; e_k^{f^T}\right]$, $A_{cli}^a = A_i^a - L_i^a C_i^a$, $W_{cli}^a = W_i^{a1} - L_i^a W_i^{a2}$, $E_j^a = \mathrm{diag}(E_j, I_{n_F}, 0_{n_f})$ and

$$A_i^a = \begin{bmatrix} A_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -I_{n_f} \end{bmatrix} \qquad L_i^a = \begin{bmatrix} L_i & 0 \\ -B_i^F & -A_i^F \\ D_i^F & C_i^F \end{bmatrix}$$

$$W_i^{a1} = \begin{bmatrix} W_i^1 & R_i^1 \\ 0 & 0 \\ 0 & I_{n_f} \end{bmatrix} \qquad H_i^a = \begin{bmatrix} H_i \\ 0 \\ 0 \end{bmatrix} \qquad T_i^{aT} = \begin{bmatrix} 0 \\ 0 \\ I_{n_f} \end{bmatrix}$$

$$C_i^a = \begin{bmatrix} C_i & 0 & 0 \\ 0 & I_{n_F} & 0 \end{bmatrix} \qquad W_i^{a2} = \begin{bmatrix} W_i^2 & R_i^2 \\ 0 & 0 \end{bmatrix}.$$

Since (21) is similar to (7) up to matrix and state vector augmentations, Theorem 1 can be adapted to determine the observer and post filter gains L_i^a such that (21) satisfies (S_3) and (S_4).

Corollary 2. *The SFO (3) and post-filter (4) for the DTSDS (1), satisfying (S3) and (S4) exist if the triplets* (E_i, A_i, C_i) *are finite dynamics detectable and impulse observable (see Koenig & Marx, 2009 for conditions) and are obtained by finding the symmetric matrices* $P_i \in \mathbb{R}^{(n+n_F+n_f)\times(n+n_F+n_f)}$, *symmetric positive definite matrices* $G_i \in \mathbb{R}^{(n+n_F+n_f)\times(n+n_F+n_f)}$, *matrices* $\tilde{G}_i \in \mathbb{R}^{(n+n_F+n_f)\times(m+n_F)}$ *and* $M_i \in \mathbb{R}^{(n+n_F+2n_f+n_d+n_\phi)\times(n+n_F+n_f)}$ *minimizing* $\bar{\gamma}_2$ *under the constraints (8b)–(8c)–(22) for* $(i,j) \in \mathbf{N}_N^2$.

$$E_i^{aT} P_i E_i^a \geq 0 \tag{22}$$

where $\bar{\gamma}_2 = \gamma_2^2$ *and*

$$\Theta_{ik} = \tilde{T}_{ik}^a + \tilde{C}_i^{aT} C_i^a G_i C_i^{aT} \tilde{C}_i^a + \mathbb{S}\left(M_i \tilde{A}_i^a + M_i C_i^{aT} \tilde{C}_i^a + \tilde{C}_i^{aT} C_i^a \tilde{G}_i \tilde{C}_i^a\right)$$

$$\tilde{T}_{ik} = \mathrm{diag}\Big(\mathrm{diag}(\beta^2 I_n, 0_{n_F+n_f}) + T_i^{aT} T_i^a - E_k^{aT} P_i E_k^a, -\bar{\gamma}_2 I_{n_d+n_f}, -I_{n_\phi}\Big)$$

$$\tilde{A}_i = \left[A_i^a \quad W_i^{a1} \quad H_i^a\right]$$

$$\tilde{C}_i = \left[C_i^a \quad W_i^{a2} \quad 0\right].$$

The observer and filter gains are given by $L_i^a = G_i^{-1}\tilde{G}_i$, *where* $G_i = \mathrm{diag}(G_i^1, G_i^2)$ *and*

$$\tilde{G}_i = \begin{bmatrix} \tilde{G}_i^1 & 0_{n\times n_F} \\ \tilde{G}_i^2 & \tilde{G}_i^3 \\ \tilde{G}_i^4 & \tilde{G}_i^5 \end{bmatrix} \tag{23}$$

with $G_i^1 \in \mathbb{R}^{n\times n}$ *and* $G_i^2 \in \mathbb{R}^{(n_F+n_f)\times(n_F+n_f)}$.

Proof. The proof is similar to the one of Theorem 1 and thus omitted. □

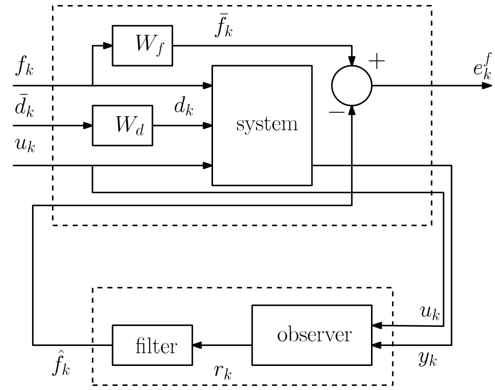


Fig. 2. Improved robust fault diagnosis scheme.

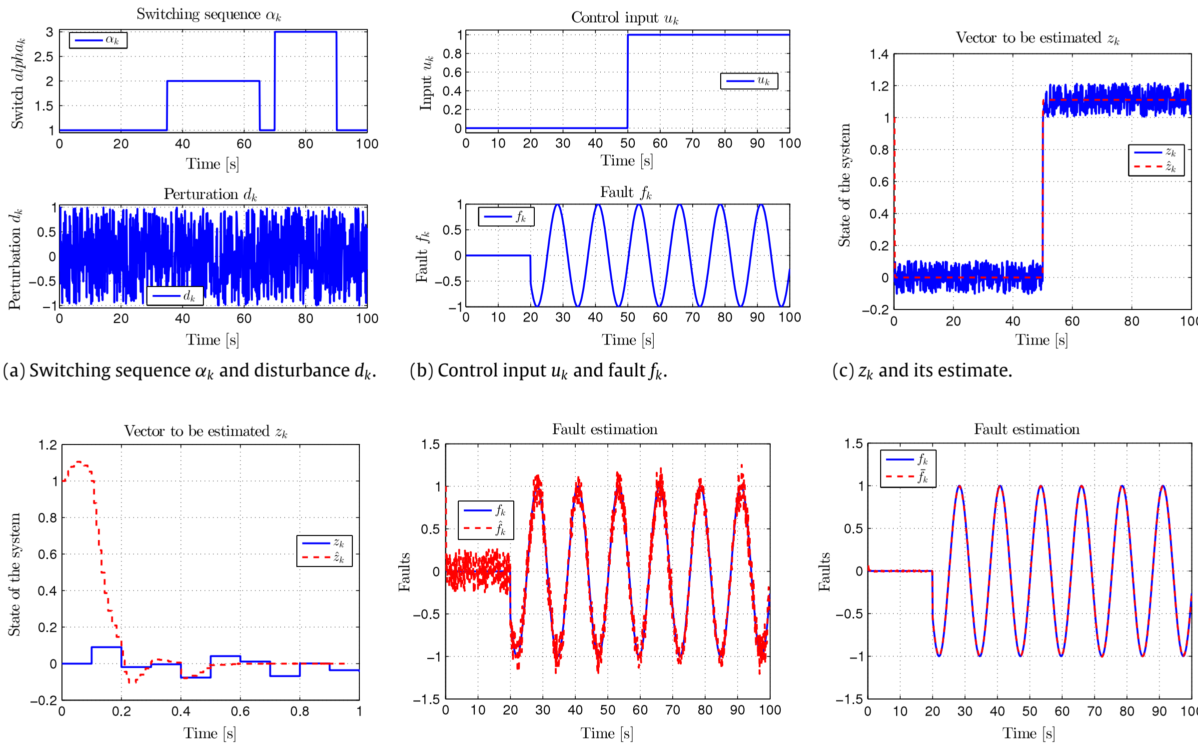
Remark 3. The null block in $\tilde{G}_i$ does not introduce any equality constraint, since it suffices to use the secondary LMI variables $\tilde{G}_i^1, \ldots, \tilde{G}_i^5$ in (23). The null block of $\tilde{G}_i$ and the block diagonal structure of G_i imply the nullity of the $(1,2)$ block of L_i^a.

Remark 4. If the main purpose is state estimation or filtering rather than fault estimation, the estimator design should be slightly modified by adding $R_i^1\hat{f}_k$ in (3a), $R_i^2\hat{f}_k$ and (3b) and by minimizing the $\mathcal{L}_2$-gain from the exogenous inputs to the estimation or filtering error e or $\tilde{z}$.

Remark 5. Analogously to standard H_∞-control and as depicted by Fig. 2, dynamical filters can be introduced in the design procedure in order to improve the fault diagnosis and avoid to impose hard constraints on the whole frequency range (see Chapter 6.5 of Blanke et al., 2006). The filter W_d imposes an attenuation level of the UI in specific frequency ranges and W_f is introduced to define the desired frequency response of $\hat{f}_k$ to the fault.

4. Numerical example

The following example illustrates the performance of the SFO (3) and the filter (4), proposed in Section 3. Consider the DTSDS (1)



(a) Switching sequence α_k and disturbance d_k. (b) Control input u_k and fault f_k. (c) z_k and its estimate.

(d) z_k: zoom on transient response. (e) Fault estimation without filters. (f) Fault estimation with W_d and W_f.

Fig. 3. Simulation results.

defined by $\phi(x_k, u_k) = 0.1\sin(x_{2k})$, $E_i = \text{diag}(0, 1)$ and

$$A_1 = \begin{pmatrix} 0.9 & 0.1 \\ 0 & 0.1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0.7 & 0.15 \\ 0 & 0.1 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} 0.5 & 0.15 \\ 0 & 0.1 \end{pmatrix}, \quad B_i = \begin{pmatrix} i \\ 0 \end{pmatrix}, \quad H_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

$$R_i^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad W_i^1 = \begin{pmatrix} 0 \\ 0.1 \end{pmatrix}, \quad C_i^T = \begin{pmatrix} 1 \\ 0.1 \end{pmatrix}, \quad T_i^T = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$D_i = 0$, $R_i^2 = 0$ and $W_i^2 = 0.3$, for $i \in \mathbf{N}_3$. The fault f_k affects the first system state and the perturbation d_k affects the second state and the measurement output y_k. One can readily verify that the system does not satisfy the UI decoupling conditions of Koenig et al. (2008). The considered perturbation is a white noise. The system inputs are presented in Fig. 3(a) and (b). The estimation results, obtained for $\hat{x}_0 = [1\ 1]^T$, are presented in Fig. 3(c) and (d). One can notice that the noise disturbance is well attenuated, and the estimator fast converges to the good value. The filter proposed in (4) is implemented in order to estimate the fault f_k. The result of the fault estimation is presented in Fig. 3(e). According to Remark 5, lowpass filters W_f and W_d^{-1} are used and the improved results presented in Fig. 3(f) are obtained. One can note that the estimation of the faults quickly converge toward the real value of the fault, despite the nonlinearities and the noise.

5. Conclusion

In this paper, a robust state and fault observer is designed for discrete-time switched nonlinear descriptor systems. This generic class of systems were not envisaged in the diagnosis framework. The design objectives are to minimize the $\mathcal{L}_2$-gain from the unknown inputs to the state and fault estimation errors. LMI conditions are obtained using switched Lyapunov functions to avoid conservatism introduced by single Lyapunov functions and filters can be introduced to improve the fault estimation robustness. The proposed approach could be extended to a wide class of systems such as LPV or descriptor Takagi–Sugeno systems.

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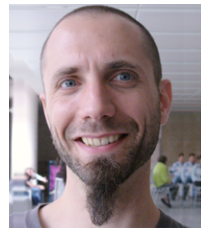

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