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H_∞-filtering and state feedback control for discrete-time switched descriptor systems

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Abstract: The problem of H_∞ filtering for a class of discrete-time switched systems with unknown inputs is investigated. By using a switched Lyapunov function, sufficient conditions for the solution of this problem are obtained in terms of linear matrix inequalities. Filtering is envisaged both with proportional and proportional integral observers. In addition, the results obtained in observer design are transposed to the controller design for switched descriptor systems. The control of a switched uncertain descriptor system is also treated. A numerical example is given to illustrate the presented results.

1 Introduction

Many natural and artificial systems and processes encompass several modes of operation with different dynamical behaviour in each mode. In practice, this phenomenon may occur when applying gain-scheduled controllers (where different controllers are designed for each operating point and controllers are switched when the operating conditions change), or with power converter systems (where the switching signal is determined by pulse with PWM modulation), or in fault diagnosis (the healthy and faulty behaviours are modelled by different subsystems; then the fault detection consists in determining the switching time [1]). To model different behaviours and the switching from one to another, the switched systems were introduced. Switched systems are defined by a collection of dynamical (linear and/or nonlinear) subsystems together with a switching rule that specifies the switching between these subsystems. A survey on the basic problems in switched system stability and design is available in [2] (and the references therein).

Recently, the controller and observer synthesis for a switched system has been extensively investigated: in [3] stability for arbitrary switching sequences and construction of stabilising switching sequences are studied. Piecewise quadratic Lyapunov functions were used in [4] to study the stability of continuous-time hybrid systems or in [5] for stability characterisation and disturbance attenuation. In [6], stability and static output feedback for discrete-time switched systems were treated with switched Lyapunov functions; [7] studied the output feedback for nonlinear switched systems; and [8] addressed the problems of stability and control of switched systems with varying delays. The average dwell time method is used in [9] to study the stability of switched discrete-time linear system affected by delays. More recently, the problem of optimal filtering was treated in [10] for a class of discrete-time switched systems with state delays with an H_∞-filtering approach, and in [11] for a class of uncertain time-delay fuzzy systems with an $L_2 - L_\infty$-filtering approach (the minimised criteria is the ratio error peak/disturbance energy). In addition, an extension of the results presented in [6] was proposed for switched uncertain systems with polytopic uncertainty [12]. However, to the best of the authors' knowledge, few results exist for the class of switched descriptor systems. Only [13], has proposed an extension of the Luenberger observer for switched descriptor systems.

In this note, an H_∞-filtering and state feedback control are developed by using switched a Lyapunov function approach for discrete-time switched descriptor systems with unknown inputs. Contrary to [13], no coordinate transformation is needed. The controller and observer design is reduced to the computation of the gains by linear matrix inequalities (LMI) optimisation. It should be pointed that, unlike [6, 14], no additional matrix variables are necessary to solve the LMI. It results in a simple design procedure for both proportional (P) and proportional integral (PI) observers, and a state feedback controller. Moreover, the state feedback control is extended to the design of a robust state feedback controller for switched uncertain systems with polytopic uncertainty. The ease of use of this method should lead to future works dedicated to the control and estimation for a more general class system like a time-delay switched descriptor system, nonlinear switched descriptor system, and so on.

This note is organised as follows. Section 2 presents the problem statement, three different problems are addressed: P observer design, PI observer design and full-state feedback control. The state estimation and H_∞-control is envisaged in Section 3. Before concluding, the PI observer performance is illustrated through a numerical example in Section 4.

Notation 1: $(.)^{\mathrm{T}}$ stands for the transpose matrix and $(*)$ is used for the blocks induced by symmetry; $(.) > 0$ denotes a symmetric positive definite matrix; I is used to denote an identity matrix with appropriate dimension; and the notation $\ell_2[0, \infty)$ refers to the space of square summable infinite vector sequences with the usual norm $\|.\|_2$.

2 Preliminaries and problem formulation

Consider the following discrete-time switched descriptor system

$$\begin{aligned}\sum_{j=1}^{N}\alpha_j(k+1)E_j x_{k+1} &= \sum_{i=1}^{N}\alpha_i(k)\big(A_i x_k + B_i u_k + W_i w_k\big)\\ y_k &= \sum_{i=1}^{N}\alpha_i(k)C_i x_k\\ z_k &= \sum_{i=1}^{N}\alpha_i(k)T_i x_k\end{aligned} \tag{1}$$

where E_j, $A_i \in \mathbb{R}^{p\times n}$, are in the general form and may be rectangular, $B_i \in \mathbb{R}^{p\times t}$, $W_i \in \mathbb{R}^{p\times q}$, $C_i \in \mathbb{R}^{m\times n}$, $T_i \in \mathbb{R}^{r\times n}$. The signal $x \in \mathbb{R}^n$ denotes the descriptor vector; $u \in \mathbb{R}^{\mathrm{T}}$ is the control input; $y \in \mathbb{R}^m$ is the measurement output; $z \in \mathbb{R}^r$ is the vector to be estimated or controlled and r satisfies $r \le n$; and $w_k \in \mathbb{R}^q$ is the disturbance input which is assumed to belong to $\ell_2[0, \infty)$. $\alpha_i(k)$ is the switching signal

$$\alpha_i : \mathbb{Z}^+ \to \{0, 1\}, \quad \sum_{i=1}^{N}\alpha_i(k) = 1,\ k \in \mathbb{Z}^+ = \{0, 1, \ldots\}$$

which specifies which subsystem is activated at each time k. For example, if $\alpha_i(k) = 1$, $\alpha_{v\neq i}(k) = 0$, $\alpha_j(k+1) = 1$ and $\alpha_{v\neq j}(k+1) = 0$, then it means that the matrices $(E_j, A_i, B_i, W_i, C_i)$ are activated. In the remaining, it is assumed that C_i, T_i are of full row rank and W_i are of full column rank.

Definition 1: A regular descriptor system (E_i, A_i, B_i, C_i) is said to be

1. Impulse-free if it exhibits no impulse behaviour;

2. Stable if all finite roots of $\det(zE_i - A_i)$ are inside the unit circle;

3. Finite dynamics detectable if there exists L_i such that $(E_i, A_i - L_iC_i)$ is regular and stable;

4. Impulse observable if there exists L_i such that $(E_i, A_i - L_iC_i)$ is regular and impulse-free;

5. Finite dynamics stabilisable if there exists K_i such that $(E_i, A_i + B_iK_i)$ is regular and stable;

6. impulse controllable if there exists K_i such that $(E_i, A_i + B_iK_i)$ is regular and impulse-free.

In the remaining, the three following problems will be addressed.

Problem 1: Consider the following proportional H_∞-observer for the switched descriptor system (1)

$$\begin{aligned}\sum_{j=1}^{N}\alpha_j(k+1)E_j\hat{x}_{k+1} &= \sum_{i=1}^{N}\alpha_i(k)\big(A_i\hat{x}_k + B_iu_k\\ &\quad + K_i^p\big(y_k - C_i\hat{x}_k\big)\big)\\ \hat{z}_k &= \sum_{i=1}^{N}\alpha_i(k)T_i\hat{x}_k\end{aligned} \tag{2}$$

where $K_i^p \in \mathbb{R}^{p\times m}$ are the gains of the observer, $\hat{x} \in \mathbb{R}^n$ is the estimate of x and $\hat{z} \in \mathbb{R}^r$ is the estimate of z. The gains K_i^p are determined such that the following specifications are guaranteed:

S_1. The state estimation error $(e_k = x_k - \hat{x}_k)$ is globally asymptotically stable and impulse-free, when $w_k = 0$;

S_2. The closed-loop transfer function $G_{\tilde{z}w}(z) = T_i\big(zE_j - \big(A_i - K_i^pC_i\big)\big)^{-1}W_i$ from the unknown input w_k to the estimation error $\tilde{z}_k = z_k - \hat{z}_k$ guarantees the

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H_∞-norm constraint $\|G_{\tilde{z}w}(z)\|_\infty < \gamma$ for some prescribed positive scalar γ.

In other words, the observer provides an asymptotical estimation of the state variables in the disturbance-free case, and a prescribed precision for the estimation of a linear combination of the state variables in the presence of disturbances.

Problem 2: Consider the following PI observer for the switched descriptor system (1)

$$\begin{aligned}\sum_{j=1}^{N}\alpha_j(k+1)E_j\hat{x}_{k+1} &= \sum_{i=1}^{N}\alpha_i(k)\big(A_i\hat{x}_k + B_iu_k + K_i^p \\ &\quad \times (y_k - C_i\hat{x}_k) + W_i\hat{w}_k\big) \\ \hat{w}_{k+1} &= \hat{w}_k - \sum_{i=1}^{N}\alpha_i(k)K_i^I(y_k - C_i\hat{x}_k) \\ \hat{z}_k &= \sum_{i=1}^{N}\alpha_i(k)T_i\hat{x}_k\end{aligned} \tag{3}$$

where the P gains K_i^p and the integral gains K_i^I are determined such that S_1, S_2 and the following specifications are guaranteed:

S_3. The estimation error $(e_k = x_k - \hat{x}_k)$ and $w_k - \hat{w}_k$ are globally asymptotically stable when w_k is constant;

S_4. $\hat{w}_k$ represents the mean value of the unknown input w when it is not constant.

In (1), w can be considered as an unknown parameter or a fault affecting the system. In this case, the augmentation of the observer dimension allows one to estimate this parameter if it is constant, or at least its mean value.

Problem 3: Consider the following H_∞-state feedback controller for the switched descriptor system (1)

$$u_k = -\sum_{i=1}^{N}\alpha_i(k)K_ix_k \tag{4}$$

where the gains $K_i \in \mathbb{R}^{p\times m}$ are determined such that the following specifications are guaranteed:

S_5. The closed-loop system $E_ix_{k+1} = (A_i - B_iK_i)x_k$ is regular, globally asymptotically stable and impulse-free, when $w_k = 0$;

S_6. The closed-loop transfer function $G_{zw}(z) = T_i\big(zE_i - (A_i - B_iK_i)\big)^{-1}W_i$ from w_k to the controlled output z_k guarantees the $H\infty$-constraint $\|G_{zw}(z)\|_\infty < \gamma$ for some prescribed positive scalar γ.

The design of a robust H_∞-state feedback controller is also envisaged for an uncertain switched descriptor system.

3 State estimation, H_∞-filtering and H_∞-state feedback control

In this section, the main results of this note concerning estimation and control are presented. On the one hand, estimation is performed in the H_∞-framework, in order to minimise the influence of the unknown input on the estimates of the state variables (or a linear combination of the state variables). On the other hand, the designed control laws are state feedback ensuring a bounded H_∞-norm of the transfer from the unknown input to the controlled output. Moreover, controller design is envisaged also for uncertain systems described by polytopic matrices.

3.1 Problem 1: P H_∞-observer design

From (1) and (2), the following equation, ruling the estimation error, is obtained

$$\begin{aligned}\sum_{j=1}^{N}\alpha_j(k+1)E_je_{k+1} &= \sum_{i=1}^{N}\alpha_i(k)\big(A_i - K_i^pC_i\big)e_k \\ &\quad + \sum_{i=1}^{N}\alpha_i(k)W_iw_k \\ \tilde{z}_k &= \sum_{i=1}^{N}\alpha_i(k)T_ie_k\end{aligned} \tag{5}$$

The observer gains are determined by studying the stability and the H_∞-norm bound of the system generating the state estimation error. The following theorem details the sufficient existence conditions of the observer and the computation of the observer gains.

Theorem 1: The switched P observer (2) for the switched descriptor system (1), guaranteeing S_1 and S_2, exists if the triplets (E_i, A_i, C_i), $i \in \varepsilon = \{1, 2, \ldots, N\}$ are both finite dynamics detectable and impulse observable, and if there exist symmetric positive definite matrices $P_i, \bar{P}_i \in \mathbb{R}^{p\times p}$ and matrices $U_i \in \mathbb{R}^{p\times m}$ (for $i = 1, \ldots, N$) satisfying the following LMIs

$$E_i^{\mathrm{T}}P_iE_i \geq 0 \quad \forall i \in \varepsilon = \{1, 2, \ldots, N\} \tag{6}$$

$$\begin{bmatrix} P_j - 2P_i & P_iA_i - U_iC_i & P_iW_i & 0 \\ * & T_i^{\mathrm{T}}T_i - E_i^{\mathrm{T}}P_iE_i - \bar{P}_i & 0 & \bar{P}_i \\ * & * & -\gamma^2 I_q & 0 \\ * & * & * & -\bar{P}_i \end{bmatrix} < 0 \quad \forall i, j \in \varepsilon \tag{7}$$

The gains of the observer are given by $K_i^p = P_i^{-1}U_i$.

Proof: To establish sufficient conditions for the existence of (2), satisfying the specifications S_1 and S_2, the following inequality should be verified [15]

$$H(e, \tilde{z}, w, k) = V_{k+1} - V_k + \tilde{z}_k^{\mathrm{T}}\tilde{z}_k - \gamma^2 w_k^{\mathrm{T}} w_k < 0 \qquad (8)$$

where $V_k = e_k^{\mathrm{T}}\left(\sum_{i=1}^{N} \alpha_i(k)E_i^{\mathrm{T}} P_i E_i\right)e_k$ is a switched parameter-dependent Lyapunov function and $P_i > 0$ are positive definite matrices. Computing the difference $V_{k+1} - V_k$, along the solution of (5), the relation (8) becomes

$$\begin{aligned} H(e, \tilde{z}, w, k) = {} & e_{k+1}^{\mathrm{T}}\left(\sum_{j=1}^{N} \alpha_j(k+1)E_j^{\mathrm{T}} P_j E_j\right)e_{k+1} - e_k^{\mathrm{T}} \\ & \times \left(\sum_{i=1}^{N} \alpha_i(k)\left(E_i^{\mathrm{T}} P_i E_i - T_i^{\mathrm{T}} T_i\right)\right)e_k - \gamma^2 w_k^{\mathrm{T}} w_k \end{aligned} \qquad (9)$$

To take into account all possible switches, we consider the case $\alpha_i(k) = 1$, $\alpha_{u\neq i}(k) = 0$ and $\alpha_j(k+1) = 1$, $\alpha_{v\neq j}(k+1) = 0$, where $i, j \in \varepsilon = \{1, 2, \ldots, N\}$. Therefore (9) and (1) are, respectively, equivalent to

$$\begin{aligned} H(e, \tilde{z}, w, k) = {} & e_{k+1}^{\mathrm{T}} E_j^{\mathrm{T}} P_j E_j e_{k+1} - e_k^{\mathrm{T}}\left(E_i^{\mathrm{T}} P_i E_i - T_i^{\mathrm{T}} T_i\right) \\ & \times e_k - \gamma^2 w_k^{\mathrm{T}} w_k \end{aligned} \qquad (10)$$

and

$$\begin{aligned} E_j x_{k+1} &= A_i x_k + B_i u_k + W_i w_k \\ y_k &= C_i x_k \\ z_k &= T_i x_k \end{aligned} \qquad (11)$$

With (10) and (11), it follows

$$\begin{aligned} H(e, \tilde{z}, w, k) = {} & e_{k+1}^{\mathrm{T}} E_j^{\mathrm{T}} P_j E_j e_{k+1} \\ & - e_k^{\mathrm{T}}\left(E_i^{\mathrm{T}} P_i E_i - T_i^{\mathrm{T}} T_i\right)e_k - \gamma^2 w_k^{\mathrm{T}} w_k \\ = {} & \left[e_k^{\mathrm{T}}\left(A_i^{\mathrm{T}} - C_i^{\mathrm{T}} K_i^{pT}\right) + w_k^{\mathrm{T}} W_i^{\mathrm{T}}\right] \\ & \times P_j\left[(A_i - K_i^p C_i)e_k + W_i w_k\right] \\ & + e_k^{\mathrm{T}}\left(T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i\right)e_k - \gamma^2 w_k^{\mathrm{T}} w_k \\ = {} & e_k^{\mathrm{T}}\Big[\left(A_i^{\mathrm{T}} - C_i^{\mathrm{T}} K_i^{p\mathrm{T}}\right)P_j\left(A_i - K_i^p C_i\right) \\ & + T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i\Big]e_k \\ & + e_k^{\mathrm{T}}\left(A_i^{\mathrm{T}} - C_i^{\mathrm{T}} K_i^{p\mathrm{T}}\right)P_j W_i w_k \\ & + w_k^{\mathrm{T}} W_i^{\mathrm{T}} P_j\left(A_i - K_i^p C_i\right)e_k \\ & + w_k^{\mathrm{T}}\left[W_i^{\mathrm{T}} P_j W_i - \gamma^2 I\right]w_k < 0 \end{aligned}$$

which can be rewritten as

$$\begin{bmatrix} e_k \\ w_k \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \beta & \left(A_i^{\mathrm{T}} - C_i^{\mathrm{T}} K_i^{p\mathrm{T}}\right)P_j W_i \\ * & -\gamma^2 I + W_i^{\mathrm{T}} P_j W_i \end{bmatrix} \begin{bmatrix} e_k \\ w_k \end{bmatrix} < 0 \qquad (12)$$

where

$$\begin{aligned} \beta = {} & (A_i - K_i^p C_i)^{\mathrm{T}} P_j (A_i - K_i^p C_i) + T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i \\ & + \bar{P}_i - \bar{P}_i \end{aligned} \qquad (13)$$

It follows that $H(e, \tilde{z}, w, k) < 0$ for any non-zero vector $\begin{bmatrix} e_k^{\mathrm{T}} & w_k^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ if

$$\begin{bmatrix} \beta & \left(A_i^{\mathrm{T}} - C_i^{\mathrm{T}} K_i^{p\mathrm{T}}\right)P_j W_i \\ * & -\gamma^2 I + W_i^{\mathrm{T}} P_j W_i \end{bmatrix} < 0 \qquad (14)$$

and using the Schur complement formula, (14) becomes

$$\begin{bmatrix} -P_j^{-1} & (A_i - K_i^p C_i) & W_i \\ * & T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i + \bar{P}_i - \bar{P}_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0$$

which is equivalent to

$$\begin{bmatrix} -P_j^{-1} & (A_i - K_i^p C_i) & W_i & 0 \\ * & T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i - \bar{P}_i & 0 & I \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\bar{P}_i^{-1} \end{bmatrix} < 0 \qquad (15)$$

Pre-multiplying (15) by $\mathrm{diag}(P_i, I_p, I_q, \bar{P}_i)$ and noticing that $\left(P_i - P_j\right)P_j^{-1}\left(P_i - P_j\right) \geq 0$ implies $-P_i P_j^{-1} P_i \leq P_j - \left(P_i + P_i\right)$, it can be deduced that (15) is implied by

$$\begin{bmatrix} P_j - 2P_i & P_i(A_i - K_i^p C_i) & P_i W_i & 0 \\ * & T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i - \bar{P}_i & 0 & \bar{P}_i \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\bar{P}_i \end{bmatrix} < 0$$

The matrix $\bar{P}_i$ is introduced in (13) and (15) in order to ensure that $T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i - \bar{P}_i$ is negative defined. □

Remark 1: The H_∞-norm bound γ can be considered as an LMI variable to be minimised. In this case, it suffices to set $\gamma^2 = \tilde{\gamma}$ in (7), and to find matrices P_i, $\bar{P}_i$ and U_i that minimise $\tilde{\gamma}$ under the constraints (6) and (7).

Remark 2: If no disturbance affects the system (i.e. $W_i = 0$) and the asymptotical stability of the state estimation error is sufficient to ensure state estimation, then the LMI (7) becomes

$$\begin{bmatrix} P_j - 2P_i & P_i A_i - U_i C_i & 0 \\ * & -E_i^{\mathrm{T}} P_i E_i - \bar{P}_i & \bar{P}_i \\ * & * & -\bar{P}_i \end{bmatrix} < 0 \qquad (16)$$

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Remark 3: For $N = 1$ (i.e. in the case of a simple linear descriptor system without switches) and $K_i^p = 0$ (i.e. stability analysis by a H_∞-norm bound condition), we have $(A_i - K_i^p C_i) = A$, $P_j = P_i = P$, $T_i = T$, $W_i = W$, $E_i = E$, and the inequalities (16) and (12) become

$$E^{\mathrm{T}} PE \geq 0$$

$$A^{\mathrm{T}} PA + T^{\mathrm{T}} T - E^{\mathrm{T}} PE - A^{\mathrm{T}} PW\left(\gamma^2 I - W^{\mathrm{T}} PW\right) \times W^{\mathrm{T}} PA < 0$$

which are equivalent to the LMIs (5a) and (5b) in [16].

3.2 Problem 2: PI H_∞-observer design

Here, the H_∞-filtering is extended to the PI observer, in order to estimate the unknown input w. From (1) and (3), the state estimation error is described by the following equation

$$\begin{aligned} \sum_{j=1}^{N} \alpha_j(k+1) E_j^a e_{k+1}^a &= \sum_{i=1}^{N} \alpha_i(k)\left(A_i^a - K_i^a C_i^a\right) e_k^a \\ &\quad + \sum_{i=1}^{N} \alpha_i(k) W_i^a w_k \\ &= \tilde{z}_k \sum_{i=1}^{N} \alpha_i(k) T_i^a e_k^a \end{aligned} \tag{17}$$

where

$$E_j^a = \begin{bmatrix} E_j & 0 \\ 0 & I_q \end{bmatrix}, \quad A_i^a = \begin{bmatrix} A_i & -W_i \\ 0 & I_q \end{bmatrix}, \quad K_i^a = \begin{bmatrix} K_i^p \\ K_i^I \end{bmatrix},$$

$$C_i^a = \begin{bmatrix} C_i & 0 \end{bmatrix}, \quad W_i^a = \begin{bmatrix} W_i \\ 0 \end{bmatrix}, \quad T_i^a = \begin{bmatrix} T_i & 0 \end{bmatrix}$$

and

$$e_k^a = \begin{bmatrix} e_k \\ \hat{w}_k \end{bmatrix}$$

For a discussion of the advantages of the PI observer, see [17].

Corollary 1: The switched PI observer (3) for the switched descriptor system (1), guaranteeing S_1, S_2, S_3 and S_4, exists if and only if the triplets $\{E_i^a, A_i^a, C_i^a\}$, $(i \in \varepsilon = \{1, 2, \ldots, N\})$ are both finite dynamics detectable and impulse observable and if there exist symmetric positive definite matrices $P_i, \bar{P}_i \in \mathbb{R}^{(p+q)\times(p+q)}$ and matrices $U_i \in \mathbb{R}^{(p+q)\times m}$ (for $i = 1, \ldots, N$) satisfying LMIs (6) and (7), where E_i, A_i, C_i, W_i, T_i are replaced by $E_i^a, A_i^a, C_i^a, W_i^a, T_i^a$, respectively.

Proof: The proof is similar to that of Theorem 1, except that the augmented matrices are used. □

3.3 Problem 3: H_∞-state feedback controller design

The state feedback controller is envisaged in two different cases: first, it is assumed that the model of the switched descriptor system is exactly known, and secondly, the control of uncertain switched descriptor systems is addressed. The uncertainty affecting the different matrices is taken into account by a polytopic representation of the matrices where the weighting parameters are unknown.

3.3.1 Switched descriptor systems without uncertainty: The objective is to determine the gains of the control law (4) to be applied to the system (1) in order that the closed-loop system defined by

$$\begin{aligned} \sum_{j=1}^{N} \alpha_j(k+1) E_j x_{k+1} &= \sum_{i=1}^{N} \alpha_i(k)\left(A_i - B_i K_i\right) x_k \\ &\quad + \sum_{i=1}^{N} \alpha_i(k) W_i w_k \\ z_k &= \sum_{i=1}^{N} \alpha_i(k) T_i x_k \end{aligned} \tag{18}$$

is stable, impulse-free and H_∞-norm bounded. In this section, that is assumed that the system is square, that is, $p = n$.

Theorem 2: The control law (4) for the switched descriptor system (1) guaranteeing S_5 and S_6 exists if and only if the triplets $\{E_i, A_i, B_i\}$, $(i \in \varepsilon = \{1, 2, \ldots, N\})$ are both finite dynamics stabilisable and impulse controllable and if there exist symmetric positive definite matrices Q_i, $\bar{Q}_i \in \mathbb{R}^{p\times p}$ and matrices $U_i \in \mathbb{R}^{\ell \times p}$, (for $i = 1, \ldots, N$) satisfying the following LMIs

$$E_i + E_i^{\mathrm{T}} - Q_i \geq 0 \quad \forall i \in \varepsilon \tag{19}$$

$$\begin{bmatrix} -Q_j & (A_i Q_i - B_i U_i) & W_i & 0 & 0 \\ * & -Q_i E_i^{\mathrm{T}} - E_i Q_i - Q_i + \bar{Q}_i & 0 & I_p & Q_i T_i^{\mathrm{T}} \\ * & * & -\gamma^2 I_q & 0 & 0 \\ * & * & * & -\bar{Q}_i & 0 \\ * & * & * & * & -I_r \end{bmatrix} < 0 \quad \forall i, j \in \varepsilon \tag{20}$$

The gains of the controller are given by $K_i = U_i Q_i^{-1}$.

Proof: To establish sufficient conditions for the existence of (4) such that the closed-loop system satisfies the specifications S_5 and S_6, a sufficient condition is the existence of a function V_k satisfying the following inequality

$$H(x, z, w, k) = V_{k+1} - V_k + z_k^{\mathrm{T}} z_k - \gamma^2 w_k^{\mathrm{T}} w_k < 0 \qquad (21)$$

where $V_k = x_k^{\mathrm{T}}\left(\sum_{i=1}^{N} \alpha_i(k) E_i^{\mathrm{T}} P_i E_i\right) x_k$ and $P_i > 0$. Computing the difference $V_{k+1} - V_k$, along the solution of the closed-loop system (18), the relation (21) becomes

$$\begin{bmatrix} x_k \\ w_k \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \beta & \left(A_i^{\mathrm{T}} - K_i^{\mathrm{T}} B_i^{\mathrm{T}}\right) P_j W_i \\ * & -\gamma^2 I + W_i^{\mathrm{T}} P_j W_i \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} < 0 \qquad (22)$$

where $\beta = \left(A_i^{\mathrm{T}} - K_i^{\mathrm{T}} B_i^{\mathrm{T}}\right) P_j (A_i - B_i K_i) + T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i + \bar{P}_i - \bar{P}_i$ and $\alpha_i(k) = 1$, $\alpha_{v \neq i}(k) = 0$, $\alpha_j(k+1) = 1$, $\alpha_{v \neq j}(k+1) = 0$, $i,\ j,\ v \in \varepsilon = \{1, 2, \ldots, N\}$.

It follows that $H(x, z, w, k) < 0$ for any non-zero vector $\begin{bmatrix} x_k^{\mathrm{T}} & w_k^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ if

$$\begin{bmatrix} -P_j^{-1} & (A_i - B_i K_i) & W_i \\ * & T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i + \bar{P}_i - \bar{P}_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \qquad (23)$$

which is equivalent to

$$\begin{bmatrix} -P_j^{-1} & (A_i - B_i K_i) & W_i & 0 \\ * & T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i - \bar{P}_i & 0 & I \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\bar{P}_i^{-1} \end{bmatrix} < 0 \qquad (24)$$

Defining the following non-singular matrices $X_i = \mathrm{diag}(I_p, P_i^{-1}, I_q, I_p)$, the LMI (24) is equivalent to

$$X_i^{\mathrm{T}} \begin{bmatrix} -P_j^{-1} & (A_i - B_i K_i) & W_i & 0 \\ * & T_i^{\mathrm{T}} T_i - E_i^{\mathrm{T}} P_i E_i - \bar{P}_i & 0 & I \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\bar{P}_i^{-1} \end{bmatrix} X_i < 0$$

or equivalently

$$\begin{bmatrix} -P_j^{-1} & (A_i - B_i K_i) P_i^{-1} & W_i & 0 \\ * & P_i^{-1} T_i^{\mathrm{T}} T_i P_i^{-1} - P_i^{-1} E_i^{\mathrm{T}} P_i E_i P_i^{-1} - P_i^{-1} \bar{P}_i P_i^{-1} & 0 & I \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\bar{P}_i^{-1} \end{bmatrix} < 0$$

defining $U_i = K_i P_i^{-1}$ and applying the Schur complement, it follows

$$\begin{bmatrix} -P_j^{-1} & (A_i P_i^{-1} - B_i U_i) & W_i & 0 & 0 \\ * & -P_i^{-1}\left(E_i^{\mathrm{T}} P_i E_i + \bar{P}_i\right) P_i^{-1} & 0 & I & P_i^{-1} T_i^{\mathrm{T}} \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\bar{P}_i^{-1} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \qquad (25)$$

Noticing that the following inequalities

$$(P_i^{-1} E_i^{\mathrm{T}} - P_i^{-1}) P_i (E_i P_i^{-1} - P_i^{-1}) \geq 0$$
$$\left(P_i^{-1} - \bar{P}_i^{-1}\right) \bar{P}_i \left(P_i^{-1} - \bar{P}_i^{-1}\right) \geq 0$$

respectively, imply

$$\begin{aligned} -P_i^{-1} E_i^{\mathrm{T}} P_i E_i P_i^{-1} &\leq -P_i^{-1} E_i^{\mathrm{T}} - E_i P_i^{-1} + P_i^{-1} \\ -P_i^{-1} \bar{P}_i P_i^{-1} &\leq -P_i^{-1} - P_i^{-1} + \bar{P}_i^{-1} \end{aligned} \qquad (26)$$

then, one can deduced that (25) is implied by

$$\begin{bmatrix} -P_j^{-1} & (A_i P_i^{-1} - B_i U_i) & W_i & 0 & 0 \\ * & -P_i^{-1} E_i^{\mathrm{T}} - E_i P_i^{-1} - P_i^{-1} + \bar{P}_i^{-1} & 0 & I & P_i^{-1} T_i^{\mathrm{T}} \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\bar{P}_i^{-1} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \qquad (27)$$

Setting $Q_i = P_i^{-1}$ and $\bar{Q}_i = \bar{P}_i^{-1}$, (20) follows. The semi-positiveness of the Lyapunov function V_k is ensured if $E_i^{\mathrm{T}} P_i E_i \geq 0$ is satisfied. To obtain LMIs in Q_i, the following inequality is used

$$\left(P_i^{-1} - E_i\right)^{\mathrm{T}} P_i \left(P_i^{-1} - E_i\right) \geq 0 \qquad (28)$$

which is equivalent to

$$E_i^{\mathrm{T}} P_i E_i \geq E_i + E_i^{\mathrm{T}} - P_i^{-1} = E_i + E_i^{\mathrm{T}} - Q_i \qquad (29)$$

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This last inequality proves that (19) implies $E_i^{\mathrm{T}} P_i E_i \geq 0$, and thus achieves the proof. □

If no unknown input affect the system, the closed-loop system is defined by

$$\sum_{j=1}^{N} \alpha_j(k+1) E_j x_{k+1} = \sum_{i=1}^{N} \alpha_i(k)(A_i - B_i K_i) x_k \tag{30}$$

and only asymptotical stability, and admissibility are needed.

Corollary 2: The closed-loop system (30) is regular, globally asymptotically stable and impulse-free if and only if the the triplets $\{E_i, A_i, B_i\}$, $(i \in \varepsilon)$ are both finite dynamics stabilisable and impulse controllable and if there exist symmetric positive definite matrices Q_i, $\bar{Q}_i \in \mathbb{R}^{p\times p}$ and matrices $U_i \in \mathbb{R}^{t\times p}$, (for $i = 1, \ldots, N$) satisfying

$$E_i + E_i^{\mathrm{T}} - Q_i \geq 0 \quad \forall i \in \varepsilon \tag{31}$$

$$\begin{bmatrix} -Q_i E_i^{\mathrm{T}} - E_i Q_i - Q_i + \bar{Q}_i & Q_i A_i^{\mathrm{T}} - U_i^{\mathrm{T}} B_i^{\mathrm{T}} & Q_i \\ * & -Q_j & 0 \\ * & * & -\bar{Q}_i \end{bmatrix} < 0 \quad \forall i, j \in \varepsilon \tag{32}$$

and the gains of the control law are given by $K_i = U_i Q_i^{-1}$.

Proof: Define $V_k = x_k^{\mathrm{T}} \sum_{i=1}^{N} \alpha_i(k) E_i^{\mathrm{T}} P_i E_i x_k$ and compute the difference $\Delta V_k = V_{k+1} - V_k$ along the trajectory of the closed-loop system (30). The closed-loop system is regular, globally asymptotically stable and impulse-free, under the arbitrary switching law, if

$$\Delta V_k = x_k^{\mathrm{T}} \left[(A_i - B_i K_i)^{\mathrm{T}} P_j (A_i - B_i K_i) - E_i^{\mathrm{T}} P_i E_i \right] x_k < 0 \quad \forall i, j \in \varepsilon$$

and the difference ΔV_k is negative definite for any $x_k \neq 0$ if

$$(A_i - B_i K_i)^{\mathrm{T}} P_j (A_i - B_i K_i) - E_i^{\mathrm{T}} P_i E_i + \bar{P}_i - \bar{P}_i < 0 \quad \forall i, j \in \varepsilon \tag{33}$$

which is equivalent to

$$\begin{bmatrix} (A_i - B_i K_i)^{\mathrm{T}} P_j (A_i - B_i K_i) - E_i^{\mathrm{T}} P_i E_i - \bar{P}_i & I \\ * & -\bar{P}_i^{-1} \end{bmatrix} < 0 \tag{34}$$

Pre-multiplying (15) by diag(P_i^{-1}, I) and using the Schur complement formula, we obtain

$$\begin{bmatrix} -P_i^{-1} E_i^{\mathrm{T}} P_i E_i P_i^{-1} - P_i^{-1} \bar{P}_i P_i^{-1} & P_i^{-1}\left(A_i^{\mathrm{T}} - K_i^{\mathrm{T}} B_i^{\mathrm{T}}\right) & P_i^{-1} \\ * & -P_j^{-1} & 0 \\ * & * & -\bar{P}_i^{-1} \end{bmatrix} < 0 \tag{35}$$

From (26), it can be deduced that (35) is implied by (32), where $Q_i = P_i^{-1}$, $\bar{Q}_i = \bar{P}_i^{-1}$ and $U_i = K_i P_i^{-1}$. □

Remark 4: In the special case of a non-descriptor system, that is with $E_i = I$, the inequality (32) is equivalent to (18) in [6] (with $S_i = Q_i$, $S_j = Q_j$ and $C_i = I$, since state feedback is envisaged here, whereas output feedback was treated in [6]).

3.3.2 Uncertain switched descriptor systems:

Here, the previous results are extended to the following uncertain switched descriptor system

$$\begin{aligned} \sum_{j=1}^{N} \alpha_j(k+1) E_j(k+1) x_{k+1} &= \sum_{i=1}^{N} \alpha_i(k)\left(\hat{A}_i(k) x_k + \hat{B}_i(k) u_k + \hat{W}_i(k) w_k\right) \\ z_k &= \sum_{i=1}^{N} \alpha_i(k) T_i x_k \end{aligned} \tag{36}$$

where $\hat{A}_i(k)$, $\hat{B}_i(k)$ and $\hat{W}_i(k)$ are polytopic matrices, described by

$$\hat{A}_i(k) = \sum_{l_1=1}^{n_{A_i}} \xi_{il_1}^{A}(k) A_{il_1}$$

$$\hat{B}_i(k) = \sum_{l_2=1}^{n_{B_i}} \xi_{il_2}^{B}(k) B_{il_2}, \quad \hat{W}_i(k) = \sum_{l_3=1}^{n_{W_i}} \xi_{il_3}^{W}(k) W_{il_3}$$

$$\alpha_i : \mathbb{Z}^+ \to \{0, 1\}, \quad \sum_{i=1}^{N} \alpha_i(k) = 1, \quad k \in \mathbb{Z}^+ = \{0, 1, \ldots\}$$

$$\sum_{l_1=1}^{n_{A_i}} \xi_{il_1}^{A}(k) = \sum_{l_2=1}^{n_{B_i}} \xi_{il_2}^{B}(k) = \sum_{l_3=1}^{n_{W_i}} \xi_{il_3}^{W}(k) = 1$$

$$\xi_{il_1}^{A}(k), \xi_{il_2}^{B}(k), \xi_{il_3}^{W}(k) > 0$$

Each uncertain matrix $\hat{A}_i(k)$, $\hat{B}_i(k)$ and $\hat{W}_i(k)$ is described by a polytope of known vertices. The positive integers n_{A_i}, n_{B_i} and n_{W_i} are the numbers of vertices of the domains in which the uncertain matrices evolve. The positive real numbers $\xi_{il_{(.)}}^{(.)}(k)$ are the time varying, unknown weighting parameters describing the evolution of the uncertain matrices.

Theorem 3: The robust control law (4) for the uncertain switched descriptor system (36) guaranteeing S_5 and S_6 exists if and only if the triplets $\left\{E_i, A_{il_1}, B_{il_2}\right\}$, $(i \in \varepsilon = \{1, 2, \ldots, N\})$ are both finite dynamics stabilisable and impulse controllable and if there exist symmetric positive definite matrices Q_i, $\bar{Q}_i \in \mathbb{R}^{p\times p}$ and matrices $U_i \in \mathbb{R}^{t\times p}$, (for $i = 1, \ldots, N$) satisfying the following LMIs

$$E_i + E_i^{\mathrm{T}} - Q_i \geq 0 \quad \forall i \in \varepsilon \tag{37}$$

$$\begin{bmatrix} -Q_j & \left(A_{il_1}Q_i - B_{il_2}U_i\right) & W_{il_3} & 0 & 0 \\ * & -Q_iE_i^{\mathrm{T}} - E_iQ_i - Q_i + \bar{Q}_i & 0 & I & Q_iT_i^{\mathrm{T}} \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\bar{Q}_i & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \quad \forall i, j \in \varepsilon \tag{38}$$

$$l_1 = 1, \ldots, n_{A_i}, \quad l_2 = 1, \ldots, n_{B_i}, \quad l_3 = 1, \ldots, n_{W_i}$$

The gains of the controller are given by $K_i = U_iQ_i^{-1}$.

Proof: Since the LMIs of (19) and (20) are affine in the system matrices, the results of Theorem 2 can be used to derive Theorem 3, and the proof is obvious and omitted. □

4 Example

To shorten the present note, only the PI observer design is presented. We consider the system (1) defined by the following matrices

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0.4 & 0.05 & 0 \\ 0 & -0.7 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.025 & 0 & 0 \\ -0.1 & -0.35 & 0 \\ 0.2 & 0 & 0.1 \end{bmatrix}$$

$$W_1 = W_2 = \begin{bmatrix} 1 & 0.4 & 0 \end{bmatrix}^{\mathrm{T}}, \quad C_1 = \begin{bmatrix} 0.29 & 0.15 & 0.2 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -0.19 & 0.17 & 0.4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

The objective is to compute the gains of the PI observer (3) in order to illustrate the performance of the estimation of the state variables of a switched descriptor system. Each of the triplets $\left\{E_i^a, A_i^a, C_i^a\right\}$, $(i \in \varepsilon)$ is both finite dynamics detectable and impulse observable and therefore we can solve the LMIs defined in Corollary 1. After some iterations, we find $\gamma = 1.4$ and

$$K_1^a = \left[\begin{array}{cc} 1.2008 & -0.2402 \\ -0.8000 & 0.1600 \\ 0.0000 & 0.1000 \\ \hline -0.0009 & 0.0002 \end{array}\right], \quad K_2^a = \left[\begin{array}{cc} -0.3196 & 0.1279 \\ 4.1955 & -1.6782 \\ -2.5544 & 1.1218 \\ \hline 0.0067 & -0.0027 \end{array}\right]$$

The observer gives a good state estimation. More precisely, the UI attenuation properties can clearly be observed on the singular value of the subsystems $(E_j^a, (A_i^a - K_i^aC_i^a), W_i^a, T_i^a)$ for $(i,j) \in \{1, 2\}$, between w to $\tilde{z}$ as given in Figs. 1 and 2.

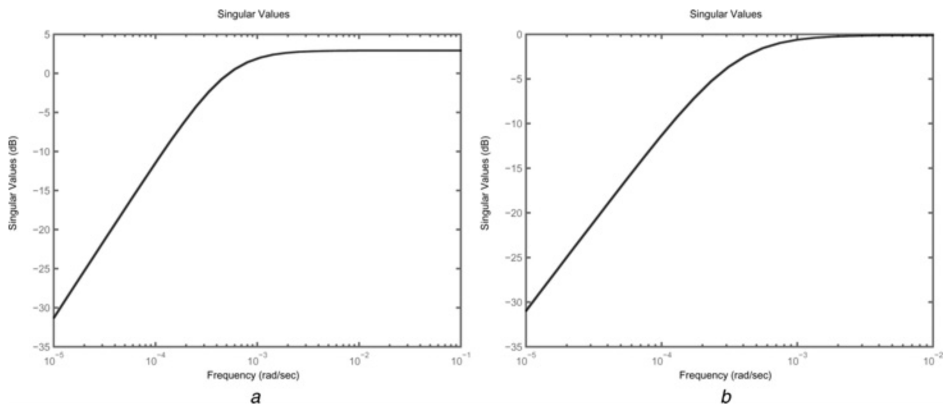

Figure 1 *Singular values of the transfer function between w to $\tilde{z}$*

a Singular value of the system $(E_1^a, (A_1^a - K_1^aC_1^a), W_1^a, T_1^a)$, between w to $\tilde{z}$
b Singular value of the system $(E_1^a, (A_2^a - K_2^aC_2^a), W_2^a, T_2^a)$ between w to $\tilde{z}$

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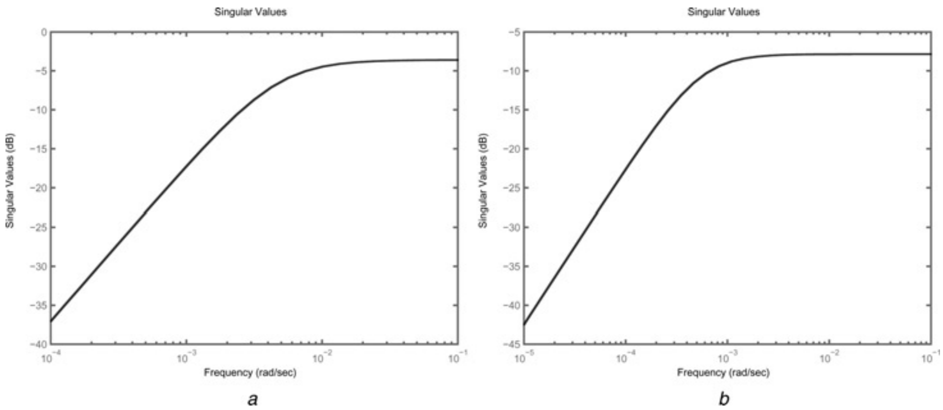

Figure 2 *Singular values of the transfer function between w to $\tilde{z}$*

a Singular value of the system (E_2^a, ($A_1^a - K_1^a C_1^a$), W_1^a, T_1^a), between w to $\tilde{z}$
b Singular value of the system (E_2^a, ($A_2^a - K_2^a C_2^a$), W_2^a, T_2^a) between w to $\tilde{z}$

5 Conclusion

The problem of H_∞-filtering and state feedback control for a class of discrete-time switched descriptor systems with unknown inputs was investigated. Sufficient conditions for the existence of H_∞-observers and a state feedback controller have been derived in terms of LMIs. The proposed approaches not only makes the resulting closed-loop system regular, causal and stable, but also guarantees a bounded H_∞-norm of the closed-loop system from the unknown input to controlled output variable. The determination of the gains of the observer (or controller) is reduced to solve a set of LMIs. Moreover, in order to improve the robustness of the proposed state feedback controller, the design method is extended to switched uncertain systems with polytopic uncertainty. Future works will be dedicated to extend the approach to nonlinear and/ or time-delay descriptor systems.

6 References

[1] RAGOT J., MAQUIN D., DOMLAN E.A.: 'Switching time estimation of piecewise linear system. Application to diagnosis'. Proc. of the 5th IFAC Symp. Fault Detection, Supervision and Safety of Technical Processes, SAFEPROCESS'2003, Washington, D.C., USA, 2003

[2] SUN Z., GE S.S.: 'Analysis and synthesis of switched linear control systems', *Automatica*, 2005, **41**, pp. 181–195

[3] LIEBERZON D., MORSE A.S.: 'Basic problems in stability and design of switching system', *IEEE Control Syst. Mag.*, 1999, **19**, (5), pp. 59–70

[4] JOHANSSON M., RANTZER A.: 'Computation of piecewise quadratic Lyapunov functions for hybridsystems', *IEEE Trans. Autom. Control*, 1998, **43**, (4), pp. 555–559

[5] LIN H., ZHAI G., ANTSAKLIS P.J.: 'Robust stability and disturbance attenuation analysis of a class of networked control systems'. Proc. 42nd IEEE Conf. Decision and Control, CDC'03, 2003, pp. 1182–1187

[6] DAAFOUZ J., RIEDINGER P., IUNG C.: 'Stability analysis and control synthesis for switched systems: a switched Lyapunov function Approach', *IEEE Trans. Autom. Control*, 2002, **47**, (11), pp. 1883–1887

[7] EL FARRAL N.H., MHASKAR P., CHRISTOFIDES P.D.: 'Output feedback control of switched nonlinear systems using multiple Lyapunov functions', *Syst. Control Lett.*, 2005, **54**, pp. 1163–1182

[8] SUN Y.G., WANG L., XIE G.: 'Delay-dependent robust stability and stabilization for discrete-time switched with mode-dependent time-varying delays', *Appl. Math. Comput.*, 2006, **180**, (2), pp. 428–435

[9] WANG R., ZHAO J.: 'Exponential stability analysis for discrete-time switched linear systems with time-delay', *Int. J. Innov. Comput. Inf. Control*, 2007, **3**, (6B), pp. 1557–1564

[10] DU D., JIANG B., SHI P., ZHOU S.: 'H_∞ filtering of discrete-time switched systems with state delays via switched Lyapunov function approach', *IEEE Trans. Autom. Control*, 2007, **52**, (8), pp. 1520–1524

[11] GONG C., SU B.: 'Robust L_2–L_∞ filtering of convex polyhedral uncertain time-delay fuzzy systems', *Int. J. Innov. Comput. Inf. Control*, 2008, **4**, (4), pp. 793–802

[12] HETEL L., DAAFOUZ J., IUNG C.: 'Stabilization of arbitrary switched linear systems with unknown time-vayring delay', *IEEE Trans. Autom. Control*, 2006, **51**, (10), pp. 1668–1674

[13] KOENIG D., MARX B., JACQUET D.: 'Unknown input observers for switched nonlinear discrete time descriptor systems', *IEEE Trans. Automat. Control*, 2008, **53**, (1), pp. 373–379

[14] FERRARI-TRECATE G., CUZZOLA F.A., MIGNONE D., MORARI M.: 'Analysis of discrete-time piecewise affine and hybrid systems', *Automatica*, 2002, **38**, (12), pp. 2139–2146

[15] BOYD S., EL GHAOUI L., FERON E., BALAKRISHNAN V.: 'Linear matrix inequalities in systems and control theory' (SIAM, Philadelphia, USA, 1994)

[16] XU S., YANG C.: 'H_∞ state feedback control for discrete singular systems', *IEEE Trans. Autom. Control*, 2000, **45**, (7), pp. 1405–1409

[17] KOENIG D.: 'Unknown input proportional multiple-integral observer design for linear descriptor systems: application to state and fault estimation', *IEEE Trans. Autom. Control*, 2005, **50**, (2), pp. 212–217

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