Technical Notes and Correspondence

Unknown Input Proportional Multiple-Integral Observer Design for Linear Descriptor Systems: Application to State and Fault Estimation

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***Abstract*—In this note, the problem of observer design for linear descriptor systems with faults and unknown inputs is considered. First, it is considered that the fault vector function f is $\bar{s}$ times piecewise continuously differentiable. If the $\bar{s}$th time derivative of f is null, then $\bar{s}$ integral actions are included into a Luenberger observer, which is designed such that it estimates simultaneously the state, the fault, and its finite derivatives face to unknown inputs. Second, when the fault is not time piecewise continuously differentiable but bounded (like actuator noise) or $\bar{s}$th time derivative of fault is not null but bounded too, a high gain observer is derived to attenuate the fault impact in estimation errors. The considered faults may be unbounded, may not be determinist, and faults and unknown inputs may affect the state dynamic and plant outputs. Sufficient conditions for the existence of such observer are given. Results are illustrated with a differential algebraic power system.**

***Index Terms*—Descriptor systems, proportional–integral (PI) observer, robustness.**

I. INTRODUCTION

Descriptor systems are very sensitive to slight input changes [4], [5] and the presence of unknown inputs (UI) is very detrimental to the design of observers. However, few results have been presented to design observers in the case of UI descriptor systems. In [7], assuming that the number of UI is strictly less than the number of measurements, a generalized Sylvester equation was used to develop a procedure for the design of a reduced-order UI observer. While in [2], an equivalent condensed form to design an UI observer was introduced, however the design procedure requires square singular systems, free UI measurements and regularity conditions.

In this note, an unknown input proportional multiple-integral observer (UIPMIO) is designed which achieves a robust state and fault estimation face to UI and bounded uncertain parameters. First, it is assumed that the fault $f(\tau)$ is of the following form:

$$f(\tau) = D_0 + D_1\tau + D_2\tau^2 + \cdots + D_{\bar{s}-1}\tau^{\bar{s}-1} \tag{1}$$

where the $\bar{s}$th time derivative of f is null (i.e., $f^{(\bar{s})} = 0$) and D_i $(i = 0, 1, 2 \ldots \bar{s} - 1)$ are unknown constant vectors. Clearly, a fault described as (1) may be unbounded.

Second, when the fault (or one component) is not time piecewise continuously differentiable but bounded or that $f^{(\bar{s})}$ is not equal to zero (or only one component) but bounded too, a high gain observer is designed, in Section III-B, that attenuates the fault impact on estimation errors. As in [7], the considered systems are in general form (i.e., rectangular). However, contrary to [7], the proposed observer allows robust state estimation in presence of parameters variations and fault estimation, filters the actuators noises and the number of UI may be up to the number of measurements. The main contributions of the note are discussed and illustrated, respectively, in Sections IV and V.

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II. PROBLEM FORMULATION AND MAIN RESULTS

Consider the linear time-invariant descriptor system

$$\begin{aligned} E^*\dot{x} &= A^*x + B^*u + F_w^*w + F_f^*f \\ y^* &= C^*x + G_w^*w + G_f^*f \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^k$, $w \in \mathbb{R}^q$, $f \in \mathbb{R}^s$, and $y^* \in \mathbb{R}^p$ denote the state vector, the control input vector, the unknown input vector, the unknown fault vector, and the output vector, respectively. E^*, $A^* \in \mathbb{R}^{m\times n}$, $B^* \in \mathbb{R}^{m\times k}$, $F_w^* \in \mathbb{R}^{m\times q}$, $F_f^* \in \mathbb{R}^{m\times s}$, $C^* \in \mathbb{R}^{p\times n}$, $G_w^* \in \mathbb{R}^{p\times q}$, and $G_f^* \in \mathbb{R}^{p\times s}$ are known constant matrices.

Let $r := \text{rank}E^* \leq n$ and without loss of generality, assume that measurements are linearly independent, i.e., $\text{rank}\,[\,C^* \quad G_w^* \quad G_f^*\,] = p$, and that UI and faults are also linearly independent, i.e.,

$$\text{rank}\begin{bmatrix} F_w^* & F_f^* \\ G_w^* & G_f^* \end{bmatrix} = q + s \tag{3}$$

with $m + p \geq q + s$. Now, since $\text{rank}E^* = r$, there exists a regular matrix P such that (2) is restricted system equivalent (r.s.e) to [7]

$$\begin{aligned} E\dot{x} &= Ax + Bu + F_w w + F_f f \\ y &= Cx + G_w w + G_f f \end{aligned} \tag{4}$$

where

$$PE^* = \begin{bmatrix} E \\ 0 \end{bmatrix} \quad PA^* = \begin{bmatrix} A \\ A_1 \end{bmatrix} \quad PB^* = \begin{bmatrix} B \\ B_1 \end{bmatrix}$$
$$PF_w^* = \begin{bmatrix} F_w \\ F_{w_1} \end{bmatrix} \quad PF_f^* = \begin{bmatrix} F_f \\ F_{f_1} \end{bmatrix}$$
$$y = \begin{bmatrix} -B_1 u \\ y^* \end{bmatrix} \in \mathbb{R}^t \quad C = \begin{bmatrix} A_1 \\ C^* \end{bmatrix} \in \mathbb{R}^{t\times n}$$
$$G_w = \begin{bmatrix} F_{w_1} \\ G_w^* \end{bmatrix} \in \mathbb{R}^{t\times q} \quad G_f = \begin{bmatrix} F_{f_1} \\ G_f^* \end{bmatrix} \in \mathbb{R}^{t\times s}$$

in which $E \in \mathbb{R}^{r\times n}$, $\text{rank}E = r$ and $t = m + p - r$.

In [7], the proposed unknown inputs proportional observer (UIPO) exists for system (4) if and only if (iff) there exists at least one disturbance free measurement and no fault f (i.e., $t > \text{rank}G_w$ and $f = 0$, see case 4.2 in Section IV). In order to relax these previous assumptions and to attenuate the bounded fault impact on estimation errors (for instance, when $f^{(\bar{s})} \neq 0$ or f is not determinist but bounded, see Section III-B), the following augmented system is considered, with $\dot{x}_I = y$:

$$\begin{aligned} \bar{E}\dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u + \bar{F}_w w + \bar{F}_f f \\ \breve{y} &= \breve{C}\bar{x} + \breve{G}_w w + \breve{G}_f f \end{aligned} \tag{5}$$

where

$$\breve{y} = \begin{bmatrix} y_I = \int_0^\tau y\,dv = C_I\bar{x} \\ y = \bar{C}\bar{x} + G_w w + G_f f \end{bmatrix} \in \mathbb{R}^{\bar{t}}$$
$$\bar{x} = [\,x^T \quad x_I^T\,]^T \in \mathbb{R}^{\bar{n}} \tag{6}$$
$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & I_t \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad \bar{F}_w = \begin{bmatrix} F_w \\ G_w \end{bmatrix}$$
$$\bar{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \in \mathbb{R}^{\bar{r}\times\bar{n}} \quad \breve{C} = \begin{bmatrix} C_I \\ \bar{C} \end{bmatrix} \in \mathbb{R}^{\bar{t}\times\bar{n}}$$
$$\bar{F}_f = \begin{bmatrix} F_f \\ G_f \end{bmatrix} \quad \breve{G}_w = \begin{bmatrix} 0 \\ G_w \end{bmatrix} \quad \breve{G}_f = \begin{bmatrix} 0 \\ G_f \end{bmatrix} \tag{7}$$
$$C_I = [\,0 \quad I_t\,] \quad \bar{C} = [\,C \quad 0\,] \in \mathbb{R}^{t\times\bar{n}}$$

and $\bar{n} = n + t, \bar{r} = r + t, \bar{t} = 2t$. Obviously

$$\text{rank}\begin{bmatrix} \bar{F}_w & \bar{F}_f \\ \breve{G}_w & \breve{G}_f \end{bmatrix} = \text{rank}\,[\,\bar{F}_w \quad \bar{F}_f\,] = q + s \tag{8}$$

iff (3) holds with $\bar{r} \geq q + s \Leftrightarrow m + p \geq q + s$.

Consider the proportional multiple-integral observer for (5) described by

$$\begin{aligned} \dot{z} &= \pi z + K_p y_I + K_{p2}\breve{y} + T\bar{B}u + \left(T\bar{F}_f - K_{p2}\breve{G}_f\right)\hat{f}_{\bar{s}} \\ \dot{\hat{f}}_{\bar{s}} &= K_I^{\bar{s}}\left(y_I - C_I\hat{\bar{x}}\right) + \hat{f}_{\bar{s}-1} \\ &\vdots \\ \dot{\hat{f}}_2 &= K_I^2\left(y_I - C_I\hat{\bar{x}}\right) + \hat{f}_1 \\ \dot{\hat{f}}_1 &= K_I^1\left(y_I - C_I\hat{\bar{x}}\right) \\ \hat{\bar{x}} &= z + N\breve{y} - N\breve{G}_f\hat{f}_{\bar{s}} \\ \hat{x} &= [\,I_n \quad 0\,]\,\hat{\bar{x}} \end{aligned} \tag{9}$$

where $z, \bar{x}, \hat{\bar{x}} \in \mathbb{R}^{\bar{n}}, \hat{x} \in \mathbb{R}^n, \hat{f}_i \in \mathbb{R}^s\ (i = 1, 2 \ldots \bar{s})$ and π, K_p, K_{p2}, T, N and $K_I^j\ (j = 1 : \bar{s})$ are constant matrices of appropriate dimensions, which must be determined such that $\hat{x}$ and $\hat{f}_i\ (i = 1, 2 \ldots \bar{s})$ asymptotically converge to x and $f^{(\bar{s}-i)}$ respectively. In other words $\hat{f}_i\ (i = 1, 2 \ldots \bar{s})$ is an estimation of the $(\bar{s} - i)$th derivative of the fault f in the form (1), which implies that $\hat{f}_{\bar{s}}$ is an estimation of f.

Lemma 1: The $\bar{n} + s\bar{s}$ th-order $UIPMIO$(9) asymptotically estimates x and $f^{(\bar{s}-i)}\ (i = 1, 2 \ldots \bar{s})$ for any initial conditions $\bar{x}(0)$, $z(0), f(0), \hat{f}_i(0)\ (i = 1, 2 \ldots \bar{s})$ and $u(t), \breve{y}(t), w(t)$ iff the following conditions hold:

1) $\tilde{A} - \tilde{K}\tilde{C}$ is Hurwitz;
2) $T\bar{E} + N\breve{C} = I_{\bar{n}}$;
3) $T\bar{F}_w = 0$;
4) $N\breve{G}_w = 0$;
5) $K_p = K_{p1} + N\breve{G}_f K_I^{\bar{s}}$;
6) $\pi = T\bar{A} - K_p C_I$;
7) $K_{p2} = \pi N$;

where

$$\tilde{A} = \begin{bmatrix} T\bar{A} & T\bar{F}_f & -N\breve{G}_f & 0 & \ldots & 0 \\ 0 & 0 & I_s & 0 & \ldots & 0 \\ 0 & 0 & 0 & I_s & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & 0 \\ & & & 0 & 0 & I_s \\ 0 & \ldots & & & 0 & 0_{s\times s} \end{bmatrix} \tag{10}$$
$$\tilde{K} = [\,K_{p1}^T \quad K_I^{\bar{s}} \quad \ldots \quad K_I^2 \quad K_I^1\,]^T$$
$$\tilde{C} = [\,C_I \quad 0 \quad \ldots \quad 0\,].$$

Proof: It can be straightforwardly deduced from following Section III-A and is omitted. ■

The unknown matrices $\tilde{K}, T, N, K_p, \pi$ and K_{p2} are deduced from algorithm 1 given in Section III-A.

III. Observer Design

The following section, is divided into two subsections. The main section is Section III-A, where under $f^{(\bar{s})} = 0$ the observer (9) for system (5) is designed and its existence and stability conditions are given. Under a bounded fault $f^{(\bar{s})}$, the observer (9) for system (5) is designed by choosing a reasonable high gain observer (which is derived from the UIPMIO), it is presented in Section III-B.

A. UIPMIO Design

Defining the fault estimation error $e_{fi} = f^{(\bar{s}-i)} - \hat{f}_i\ (i = 1, 2 \ldots \bar{s})$ and the state estimation error $\bar{e} = \bar{x} - \hat{\bar{x}}$. Assume that (2) and (4) in Lemma 1 hold true, then when the observer (9) is applied to system (5) the state estimation error $\bar{e}$ becomes

$$\bar{e} = T\bar{E}\bar{x} - z - N\breve{G}_f e_{f_{\bar{s}}}. \tag{11}$$

From (11), (5), and (9), the following estimation error dynamics are obtained:

$$\begin{aligned} \dot{\bar{e}} = &\pi\bar{e} + \left(T\bar{A} - \pi T\bar{E} - K_p C_I - K_{p2}\breve{C}\right)\bar{x} \\ &+ \pi N\breve{G}_f e_{f_{\bar{s}}} + \left(T\bar{F}_f - K_{p2}\breve{G}_f\right)e_{f_{\bar{s}}} \\ &- N\breve{G}_f\dot{e}_{f_{\bar{s}}} + \left(T\bar{F}_w - K_{p2}\breve{G}_w\right)w \end{aligned} \tag{12}$$

$$\dot{e}_{f_{\bar{s}}} = f^{(\bar{s}-(\bar{s}-1))} - \dot{\hat{f}}_{\bar{s}} = -K_I^{\bar{s}}C_I\bar{e} + e_{f_{\bar{s}-1}} \tag{13}$$

$$\begin{aligned} \dot{e}_{f_{\bar{s}-1}} &= f^{(\bar{s}-(\bar{s}-2))} - \dot{\hat{f}}_{\bar{s}-1} = -K_I^{\bar{s}-1}C_I\bar{e} + e_{f_{\bar{s}-2}} \\ &\vdots \\ \dot{e}_{f_2} &= f^{(\bar{s}-1)} - \dot{\hat{f}}_2 = -K_I^2 C_I\bar{e} + e_{f_1} \\ \dot{e}_{f_1} &= f^{(\bar{s})} - \dot{\hat{f}}_1 = -K_I^1 C_I\bar{e} \end{aligned} \tag{14}$$

since $f^{(\bar{s})} = 0$.

Substitute (13) in (12) and from conditions 2)–7) of Lemma 1, the estimation error dynamic (12) becomes

$$\dot{\bar{e}} = \left(T\bar{A} - K_{p1}C_I\right)\bar{e} + T\bar{F}_f e_{f_{\bar{s}}} - N\breve{G}_f e_{f_{\bar{s}-1}}. \tag{15}$$

Let $\tilde{e} = [\,\bar{e}^T \quad e_{f_{\bar{s}}}^T \quad e_{f_{\bar{s}-1}}^T \quad \ldots \quad e_{f_2}^T \quad e_{f_1}^T\,]^T$ from (10), (15), (13), and (14), it comes

$$\dot{\tilde{e}} = \left[\tilde{A} - \tilde{K}\tilde{C}\right]\tilde{e}. \tag{16}$$

The system dynamics $\left[\tilde{A} - \tilde{K}\tilde{C}\right]$ can be stabilized by selecting the gain $\tilde{K}$ thanks to the detectability of the pair $(\,\tilde{A}, \quad \tilde{C}\,)$.

In the sequel, it is shown how to find matrix $[\,T \quad N\,]$ such that constraints 2)–4) of lemma 1 are satisfied. For that, rewrite 2)–4) of lemma 1 in an augmented matrix equation as

$$[\,T \quad N\,]\,\Theta = \Omega \tag{17}$$

where $\Theta = \begin{bmatrix} \bar{E} & \bar{F}_w & 0 \\ \breve{C} & 0 & \breve{G}_w \end{bmatrix}$ and $\Omega = [\,I_{\bar{n}} \quad 0 \quad 0\,]$. A solution of (17) exists if [15] rank $\begin{bmatrix} \Theta \\ \Omega \end{bmatrix}$ = rankΘ which is equivalent to

$$\text{rank}\begin{bmatrix} \bar{E} & \bar{F}_w & 0 \\ \breve{C} & 0 & \breve{G}_w \end{bmatrix} = \bar{n} + \text{rank}\bar{F}_w + \text{rank}\breve{G}_w. \tag{18}$$

Then, under (18), the general solution of (17) is

$$[\,T \quad N\,] = \Omega\Theta^+ + Z\left(I_{\bar{r}+\bar{t}} - \Theta\Theta^+\right) \tag{19}$$

where Θ^+ is the generalized inverse matrix of Θ and Z is an arbitrary matrix, fixed by the designer such that the matrix T is of maximal rank, i.e., $m + p - q$ (for details, see [6]).

Now, for (5), sufficient conditions for the existence of the observer (9) according to Lemma 1 can be given.

Theorem 1: Under (8) and rank$T = m + p - q$, there exists an UIPMIO (9) for system (5) satisfying conditons 1)–7) of Lemma 1 iff the following existence conditions are fulfilled:

(18) and

$$\text{rank}\begin{bmatrix} pI_{\bar{n}} - T\bar{A} & -T\bar{F}_f & N\breve{G}_f & 0 & \dots & 0 \\ 0 & pI_s & -I_s & 0 & & \vdots \\ & 0 & pI_s & -I_s & \ddots & \\ & & 0 & \ddots & \ddots & 0 \\ \vdots & & & \ddots & \ddots & -I_s \\ 0 & & & & 0 & pI_s \\ C_I & 0 & \dots & & 0 & 0 \end{bmatrix} = \bar{n} + s\bar{s}\ \forall\, \mathbb{R}(p) \geq 0. \tag{20}$$

Proof: It is straightforwardly deduced from the above observer design and is omitted. ■

Conditions of Theorem 1 can be given directly using the matrices of the original system (2) by the following lemma.

Lemma 2: Under (3) there exists for system (2) an UIPMIO (9) according to Lemma 1 iff the following existence conditions are fulfilled:

$$\text{rank}\begin{bmatrix} E^* & A^* & F_w^* & 0 \\ 0 & E^* & 0 & F_w^* \\ 0 & C^* & G_w^* & 0 \\ 0 & 0 & 0 & G_w^* \end{bmatrix} = n + \text{rank}\begin{bmatrix} E^* & F_w^* \\ 0 & G_w^* \end{bmatrix} + \text{rank}\begin{bmatrix} F_w^* \\ G_w^* \end{bmatrix} \tag{21}$$

$$\text{rank}\begin{bmatrix} pE^* - A^* & -F_f^* & -F_w^* \\ 0 & pI_s & 0 \\ C^* & G_f^* & G_w^* \end{bmatrix} = n + \text{rank}\begin{bmatrix} F_f^* & F_w^* \\ G_f^* & G_w^* \end{bmatrix} \quad \forall\, \mathbb{R}(p) \geq 0. \tag{22}$$

Proof: See the Appendix. ■

1) Recall that (21) was established in [9]. For $E^* = I_n$, (21) generalizes the UI decoupled condition generally assumed in UI observer design, in fact (21) can be rewritten as

$$\text{rank}\begin{bmatrix} I_n & 0 & 0 \\ C^* & -I_p & 0 \\ 0 & 0 & I_q \end{bmatrix}\begin{bmatrix} I_n & 0 & F_w^* \\ C^* & G_w^* & 0 \\ 0 & 0 & G_w^* \end{bmatrix} = n + \text{rank}G_w^* + \text{rank}\begin{bmatrix} F_w^* \\ G_w^* \end{bmatrix}$$

$$\Leftrightarrow \text{rank}\begin{bmatrix} G_w^* & C^*F_w^* \\ 0 & G_w^* \end{bmatrix} = \text{rank}G_w^* + \text{rank}\begin{bmatrix} F_w^* \\ G_w^* \end{bmatrix}$$

which is equivalent to condition (24) in [10].

2) For $F_w^* = G_w^* = G_f^* = 0$, (22) generalizes the R-detectability condition (10) in [11].

The procedure for designing the UIPMIO can be now summarized.

Algorithm 1: If conditions (3), (21), and (22) hold, then an UIPMIO (9) for system (2) or (5) exists. First, system (2) is transformed into (5) and $[T \quad N]$ is computed from (19). Since the pair $(\tilde{A}, \quad \tilde{C})$ is detectable, the observer gain $\tilde{K}$ (i.e., $K_{p_1}, K_I^{\bar{s}}, K_I^{\bar{s}-1}, \dots K_I^1$) is determined such that $\tilde{A} - \tilde{K}\tilde{C}$ is Hurwitz and from conditions 5)–7) of Lemma 1, K_p, π and K_{p_2} can be respectively deduced.

B. High-Gain Observer Design

Under the hypothesis that the $\bar{s}$th time derivative of fault f is bounded (i.e., $\left|f^{(\bar{s})}\right| \leq \gamma I_s$, where $\gamma \in \Re^1$) a high gain UIPMIO (9) that attenuates the $f^{(\bar{s})}$ fault impact in the estimation error (23) can be designed for system (5).

In this case, the estimation errors (15), (13), and (14) become

$$\dot{\tilde{e}} = \left[\tilde{A} - \tilde{K}\tilde{C}\right]\tilde{e} + \tilde{F}_f f^{(\bar{s})} \tag{23}$$

where $\tilde{A}$, $\tilde{K}$, $\tilde{C}$ are defined by (10) and

$$\tilde{F}_f = \begin{bmatrix} 0_{(\bar{n}+s(\bar{s}-1))\times s} \\ I_s \end{bmatrix}.$$

Since the fault term $f^{(\bar{s})}$ in (23) is not affected by the observer gain a reasonably high gain $\tilde{K}$, (or only K_I^1) such that the stabilizing term prevails over the fault term $\tilde{F}_f f^{(\bar{s})}$, can be chosen as it will be shown in the illustrative example (see Section V). More precisely, let $\tilde{K} = \rho\tilde{K}_0$ with $\rho \in \Re^1$, then (23) becomes

$$\frac{\dot{\tilde{e}}}{\rho} = \frac{\tilde{A}}{\rho}\tilde{e} - \tilde{K}_0\tilde{C}\tilde{e} + \frac{\tilde{F}_f}{\rho}f^{(\bar{s})}. \tag{24}$$

Under a bounded $f^{(\bar{s})}$ and an observable pair $(\tilde{A}, \tilde{C})$, it exists $\tilde{K}$ such that (23) and (24) are stable. Therefore, with a high gain $\tilde{K}$ (i.e., $\rho \to \infty$), it follows that (24) can be approximated by

$$\tilde{C}\tilde{e} = 0. \tag{25}$$

Differentiating (25) and using (23), it comes

$$\tilde{C}\tilde{A}\tilde{e} = 0$$

since $\tilde{C}\tilde{e} = 0$ and $\tilde{C}\tilde{F}_f = 0$ (by construction). In same way, under the observability of the pair $(\tilde{A}, \quad \tilde{C})$, $\tilde{C}\tilde{A}^i\tilde{e} = 0$, for $i = 2 \dots \bar{n} + s\bar{s} - 1$. It follows that $\tilde{e} \to 0$.

In addition, when some parameters in model (2) are uncertain, it is well known that they can be summarized as UI and/or faults acting on the system (see case 5.1 in the following example). Since the UI do not affect the state estimation error and the fault term is not affected by the observer gain, it follows that the proposed high-gain observer is robust in presence of uncertain bounded parameters.

The procedure for designing the high-gain observer (9) for systems (2) or (5) can be summarized by algorithm 1, where the reasonable high gain $\tilde{K}$ (or only K_I^1) is chosen a posteriori by the designer. Even if, one or more components of f are not time piecewise continuously differentiable but bounded, the proposed observer can always be implemented. This is illustrated later in Section V, case 5.2.

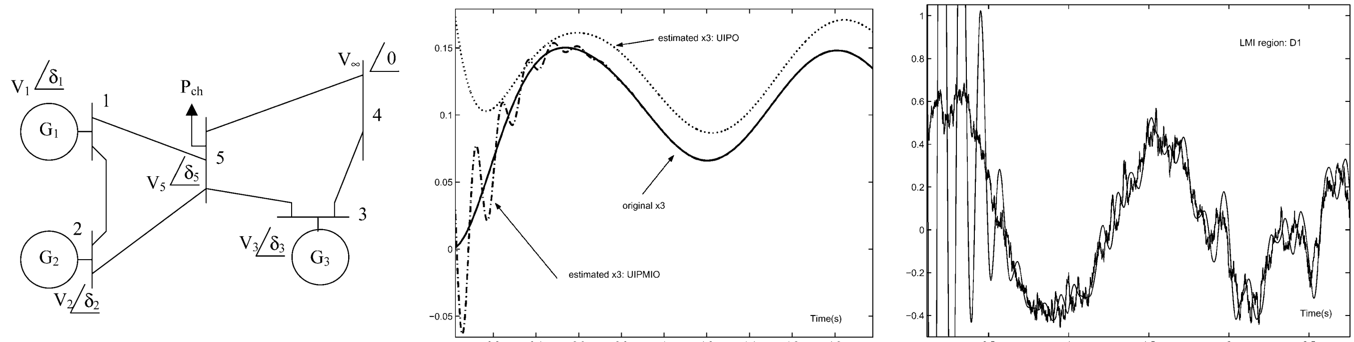
IV. Discussion

Generally speaking proportional integral observers present many advantages: robustness face of uncertain parameters [11]; accurate parameter estimation [13]; loop transfer recovery (LTR) properties with exact recovering for $\tau \longrightarrow \infty$ [12]; fault detection with disturbance rejection in steady state [14]. In the following, the performances of the proposed observer is compared with those obtained with classical observers in different cases.

Case 4.1: Classical proportional integral observer [11].

♦ Considering the following observer for system (4):

$$\begin{aligned} \dot{z} &= \pi z + K_p y + TBu + TF_f\hat{f} \\ \dot{\hat{f}} &= K_I\,(y - C\hat{x}) \\ \hat{x} &= z + Ny - NG_f\hat{f}. \end{aligned} \tag{26}$$



(a) Three machine infinite bus (b) Case 5.1: Estimation of x_3 by UIPO and UIPMIO (c) Case 5.2: Estimation of x_6 by UIPMIO

Fig. 1. System configuration and state estimation with the dynamic of the observer fixed in the LMI region $\bar{D}_1$.

Defining both state and fault estimation error $e = x - \hat{x}$ and $e_f = f - \hat{f}$, respectively. Setting $I_n - NC = TE$, $NG_w = 0$ and $TF_w = 0$, the estimation error dynamics becomes

$$\begin{aligned} \dot{e} &= \pi e + \cdots - K_p G_w w - K_p G_f f \\ \dot{e}_f &= -K_I C e - K_I G_w w - K_I G_f f + \dot{f}. \end{aligned} \tag{27}$$

From (27), it can be seen that both UI w and fault f are multiplicated by the gains K_p and K_I. Then with high gains, both UI and fault amplify the estimation error (for details, see [1, Sec. III]). Otherwise, if gains K_p and K_I are chosen small in order to attenuate both UI and fault impact, then the convergence properties of the estimation error are affected. So, under a restricted domain, estimation error tends toward zero iff $G_w = G_f = 0$ (i.e., free measurements) and $\dot{f} = 0$ (i.e., constant faults).

Comparing (23) and (27), it can be seen that the fault term $f^{(\bar{s})}$ in (23) is not affected by the observer gain. Therefore, when the fault f has the form of (1) under a bounded fault $f^{(\bar{s})}$, the proposed UIPMIO estimates simultaneously the state x, the fault f and its finite derivatives $f^{(i)}$, $(i = 1 : \bar{s} - 1)$. More precisely the fault impact $f^{(\bar{s})}$ on estimation error $\bar{e}, e_{f_{\bar{s}}}, e_{f_{\bar{s}-1}}, \ldots, e_{f_1}$ is attenuated by chosen a reasonable high gain $\tilde{K}$. In addition, if one or more components of f are not time piecewise continuously differentiable but bounded, the high gain UIPMIO (9) estimates the state x, the fault f and finite derivatives $f^{(i)}$ which is illustrated in Section V, case 5.2.

Case 4.2: Unknown inputs proportional observer [7].

♦ For $q_1 = \text{rank} G_w$ with $q_1 \leq q < t$, system (4) can be rewritten as [7]:

$$\begin{aligned} E\dot{x} &= \Phi x + Bu + F_{11} y_1 + F_{12} w_2 + (F_f - F_{11} G_{f_1}) f \\ y_1 &= C_{11} x + w_1 + G_{f_1} f \\ y_2 &= C_{12} x + G_{f_2} f \end{aligned} \tag{28}$$

where $[G_{f_1}^T \quad G_{f_2}^T]^T = UG_f$. As with case 4.1, the observer gains L_1 and L_2 (which depend on stabilizing gain Z) are affected by the fault term f in the dynamic state estimation error [7]. Otherwise for $G_f = 0$ (i.e., $G_{f_1} = G_{f_2} = 0$) and $F_f \neq 0$, the associated residual $r = \hat{y}_2 - y_2 = C_{12} e = C_{12} M(z - TEx)$ in [7] is necessarily decoupled of the fault $F_f f$ since $C_{12} M = 0$, thus a fault f affecting directly the dynamic state can not be detected.

♦ For nonsingular G_w ($\Rightarrow q = t$), the UIPO (4) proposed in [7] for system (28) cannot be designed since measurement y_2 is always disturbed by the UI (the same for residual r).

V. Application

Based on [8] and [16], a machine infinite bus system shown in Fig. 1(a) will be used to illustrate the estimation performances of the proposed observers. The dynamic behavior of the system is governed by the swing equations of the three machines G_1, G_2 and G_3. The fifth node introduces the algebraic behavior. The corresponding linear model is described as follows:

$$\begin{aligned} \dot{x}_1 &= x_1 \quad \dot{x}_2 = x_2 \quad \dot{x}_3 = x_6 \\ \dot{x}_4 &= \frac{1}{M_1}(u_1 - Y_{12} V_1 V_2 (x_1 - x_2)) \\ &\quad - \frac{1}{M_1}(Y_{15} V_1 V_5 (x_1 - x_7) + D_1 x_4) \\ \dot{x}_5 &= \frac{1}{M_2}(u_2 - Y_{21} V_2 V_1 (x_2 - x_1)) \\ &\quad - \frac{1}{M_2}(Y_{25} V_2 V_5 (x_2 - x_7) + D_2 x_5) \\ \dot{x}_6 &= \frac{1}{M_3}(u_3 - Y_{34} V_3 V_\infty x_3) \\ &\quad - \frac{1}{M_3}(Y_{35} V_3 V_5 (x_3 - x_7) + D_3 x_6) \\ 0 &= P_{ch} - Y_{51} V_5 V_1 (x_7 - x_1) - Y_{52} V_5 V_2 (x_7 - x_2) \\ &\quad - Y_{53} V_5 V_3 (x_7 - x_6) - Y_{54} V_5 V_\infty x_7 \end{aligned} \tag{29}$$

where $x_1 = \delta_1$, $x_2 = \delta_2$, $x_3 = \delta_3$, and $x_7 = \delta_5$ are the generator angles and $x_4 = \omega_1$, $x_5 = \omega_2$, and $x_6 = \omega_3$ are the generator speeds. The mechanical power $u_1 = P_1$, $u_2 = P_2$, and $u_3 = P_3$ have the same values $P_1 = P_2 = P_3 = 0.1$ pu and the nominal values of inertia M_1, M_2, M_3, of damping D_1, D_2, and D_3, of admittance Y_{15}, Y_{25}, Y_{35}, Y_{34}, and Y_{45} and of potential V_1, V_2, V_3, V_∞, and V_5 are shown in

$M_1 = 0.014$	$M_2 = 0.026$	$M_3 = 0.02$	$D_1 = 0.057$
$D_2 = 0.15$	$D_3 = 0.11$	$Y_{15} = 0.5$	$Y_{25} = 1.2$
$Y_{35} = 0.8$	$Y_{45} = 1$	$Y_{34} = 0.7$	$Y_{12} = 1.$
$V_i = 1$	$i = 1, 2, 3, \infty, 5$		

It is assumed that the only available measurements are the generator angles δ_1, δ_2, δ_3, and δ_5. In the sequel, for each UIPMIO design, the matrix $Z = 0$ and the dynamic of each observer is defined in a LMI region $\bar{D}_i = \{a_i + jb \in C,\ i = 1 : 3\}$ with [3]

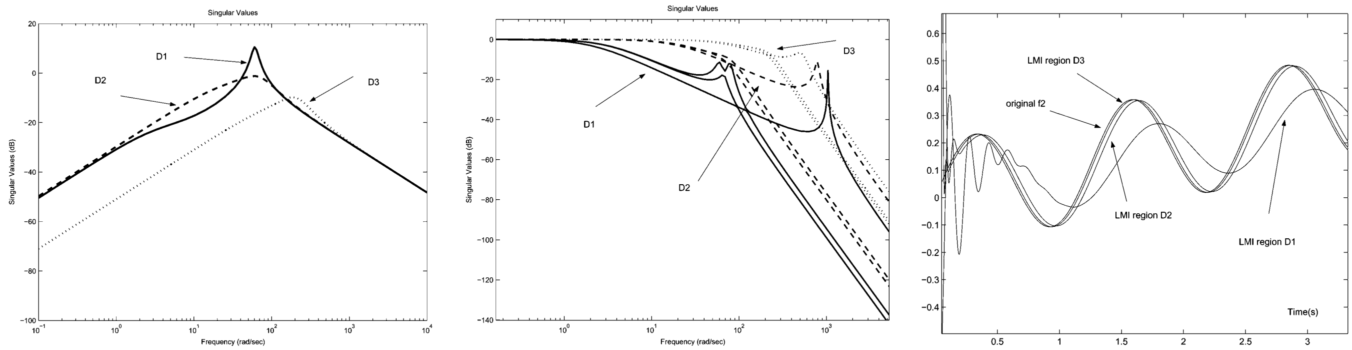
$$\{-10 < a_1 < -2.5\} \quad \{-50 < a_2 < -25\} \quad \{-100 < a_3 < -80\}.$$

Case 5.1: Parameters variation and UI.

Considering system (29) affected by the unknown load $P_{ch} = 0.2 \sin 5t$ and an uncertain admittance

$$Y_{ij} = Y_{ij_0} + \triangle Y_{ij} \tag{30}$$

where $\triangle Y_{ij} = \delta_{ij} \sin(\omega_{ij} t)$ and $|\delta_{ij}| < 0.3$, $|\omega_{ij}| < 2$ rd/s, $i = 1 : 5$, $j = 1 : 5$. The model (2) is also described by $w = P_{ch}$ and $F_f^* f = \begin{bmatrix} 0 \\ I_4 \end{bmatrix} \triangle A^* x$ with $\bar{s} = 0$. Fig. 1(b) shows that the UIPMIO proposed is unbiased although the UIPO [7] is biased.



(a) Transfer functions from fault f_2 to estimation error e_5 (b) Transfer functions from fault f to the fault actuator estimation $\hat{f}$ (c) Estimation of f_2

Fig. 2. Bode transfert functions and fault estimation performance with the dynamic of the UIPMIO fixed in the LMI region $\bar{D}_1$, $\bar{D}_2$ and $\bar{D}_3$ respectively.

Case 5.2: Simultaneously constant fault actuator f_1, unbounded nonlinear fault actuator f_2, normally distributed random fault actuator f_3 and UI.

The model (2) is also described by $w = P_{ch}$, $F_f^* = B^*$, $f(\tau) = (\,f_1 \quad f_2 \quad f_3\,)^T$ with $f_1 = \begin{cases} 0, & \text{if } \tau < 2 \\ 0.1, & \text{else} \end{cases}$, $f_2 = 0.1\tau + 0.2\sin 5\tau$, $f_3 = \begin{cases} \text{mean} = 0 \\ \text{variance} = 0.1 \end{cases}$, and $\bar{s} = 2$. A satisfactory estimation is obtained even if constant fault actuator, nonlinear unbounded fault actuator and normally distributed random fault actuator occur simultaneously. Fig. 1(c) shows that the state is well filtered. In addition, the fault attenuation properties can clearly be observed in bode transfer function given in Fig. 2(a) while Fig. 2(b) and (c) show that when the observer gain (or bandwidth of the observer) is increased the steady-state fault estimation error decreases.

VI. Conclusion

The existence conditions of a full-order nonsingular UIPMIO for descriptor systems subject to fault and UI have been given and proved. The proposed UIPMIO rejects or reduces the estimate errors of the states and the faults of the system. More precisely, if fault f has the form of (1) under the constraint that the $\bar{s}$th derivative of fault f is null, the proposed UIPMIO estimates simultaneously the state x, the fault f and its finite derivatives $f^{(i)}$, $(i = 1 : \bar{s} - 1)$ for all UI $w(t)$. Else if the $\bar{s}$th derivative of the fault f is not null (i.e., $f^{(\bar{s})} \neq 0$), a high gain UIPMIO is designed in order to attenuate the impact of $f^{(\bar{s})}$ in the estimation errors. The existence conditions of the proposed observers generalize those adopted in [7] for the design of UIPO of free fault descriptor systems. The proposed observers are robust face to parameters variations and in addition they filter the noises. The estimation performances of the proposed observers has been compared with those obtained with classical observers in different cases. Results have been illustrated in simulation.

Appendix

In the sequel, it is proved that the two sets of existence conditions stated in Theorem 1 and Lemma 2 are equivalent.

Proof: $(21) \Leftrightarrow (18)$. For that, the following equivalence

a) $$(21) \Leftrightarrow \text{rank}\begin{bmatrix} E & 0 & F_w \\ C & G_w & 0 \\ 0 & 0 & G_w \end{bmatrix} = n + \text{rank}G_w + \text{rank}\begin{bmatrix} F_w \\ G_w \end{bmatrix} \tag{31}$$

b) $(31) \Leftrightarrow (18)$

are, respectively, proved.

a) Define the regular matrices
$$P_1 = \begin{bmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & I_{2p} \end{bmatrix} \quad P_2 = \begin{bmatrix} P & 0 \\ 0 & I_p \end{bmatrix}$$
then
$$(21) \Leftrightarrow \text{rank}P_1 \begin{bmatrix} E^* & A^* & F_w^* & 0 \\ 0 & E^* & 0 & F_w^* \\ 0 & C^* & G_w^* & 0 \\ 0 & 0 & 0 & G_w^* \end{bmatrix} = n + \text{rank}P_2 \begin{bmatrix} E^* & F_w^* \\ 0 & G_w^* \end{bmatrix} + \text{rank}P_2 \begin{bmatrix} F_w^* \\ G_w^* \end{bmatrix} \Leftrightarrow (31)$$
since E is of full-row rank.

b) Substituting matrices (7) and $\bar{n} = n + t$ in (18), (18)$\Leftrightarrow$ (31) is directly obtained. ■

Proof: $(22) \Leftrightarrow (20)$. For that, the following equivalence:

1) $$(22) \Leftrightarrow \text{rank}\begin{bmatrix} pE - A & -F_f & -F_w \\ 0 & pI_s & 0 \\ -C & -G_f & -G_w \end{bmatrix} = n + \text{rank}\begin{bmatrix} F_f & F_w \\ G_f & G_w \end{bmatrix} \quad \forall\, \mathbb{R}(p) \geq 0 \tag{32}$$

2) $$(32) \Leftrightarrow \text{rank}\begin{bmatrix} p\bar{E} - \bar{A} & -\bar{F}_f & -\bar{F}_w \\ 0 & pI_s & 0 \\ \breve{C} & \breve{G}_f & \breve{G}_w \end{bmatrix} = \bar{n} + \text{rank}\,[\,\bar{F}_f \quad \bar{F}_w\,] \tag{33}$$

3) $(33) \Leftrightarrow (20)$

are, respectively, proved.

1) Define the nonsingular matrix
$$P_3 = \begin{bmatrix} P & 0 & 0 \\ 0 & I_s & 0 \\ 0 & 0 & -I_t \end{bmatrix}$$
then
$$(22) \Leftrightarrow \text{rank}P_3 \begin{bmatrix} pE^* - A^* & -F_f^* & -F_w^* \\ 0 & pI_s & 0 \\ C^* & G_f^* & G_w^* \end{bmatrix} = n + \text{rank}P_2 \begin{bmatrix} F_f^* & F_w^* \\ G_f^* & G_w^* \end{bmatrix} \ \forall\, \mathbb{R}(p) \geq 0 \Leftrightarrow (32)$$

2) Substitute matrices (7) and $\bar{n} = n + t$ in (33), it comes immediately that (33)$\Leftrightarrow$(32).

3) Define the following condition:

$$\operatorname{rank}\begin{bmatrix} pI_{\bar{n}} - T\bar{A} & -T\bar{F}_f + pN\breve{G}_f \\ 0 & pI_s \\ C_I & 0 \end{bmatrix} = \bar{n} + \operatorname{rank}\bar{F}_f \quad \forall\, \mathbb{R}(p) \geq 0 \tag{34}$$

which is equivalent to

$$\begin{aligned} &\operatorname{rank}\begin{bmatrix} -T\bar{A} & -T\bar{F}_f \\ C_I & 0 \end{bmatrix} = \bar{n} + \operatorname{rank}\bar{F}_f, \qquad p = 0 \\ &\text{and } \operatorname{rank}\begin{bmatrix} pI_{\bar{n}} - T\bar{A} \\ C_I \end{bmatrix} = \bar{n} \qquad \forall\, \mathbb{R}(p) > 0. \end{aligned} \tag{35}$$

Since (35) is obviously equivalent to (20), the problem is reduced to prove that (33) is equivalent to (34).

From (18), matrix $\begin{bmatrix} \bar{E} & \bar{F}_w \\ \breve{C} & 0 \end{bmatrix}$ is of full column rank, hence there exists a full-row rank matrix $\begin{bmatrix} T & N \\ V_1 & V_2 \end{bmatrix}$ with T of maximal rank (i.e., $m + p - q$) such that

$$\begin{bmatrix} T & N \\ V_1 & V_2 \end{bmatrix}\begin{bmatrix} \bar{E} & \bar{F}_w \\ \breve{C} & 0 \end{bmatrix} = \begin{bmatrix} I_{\bar{n}} & 0 \\ 0 & I_q \end{bmatrix}. \tag{36}$$

Let $[\,V_{21} \quad V_{22}\,] = V_2$ and define the full-row rank matrix

$$P_4 = \begin{bmatrix} T & N & 0 & 0 & 0 \\ V_1 & V_2 & 0 & -pV_{21} & -pV_{22} \\ 0 & 0 & I_s & 0 & 0 \\ 0 & 0 & 0 & I_t & 0 \end{bmatrix}$$

then, from (36), it comes

$$\begin{aligned} (33) \Leftrightarrow \operatorname{rank}P_4 &\begin{bmatrix} p\bar{E} - \bar{A} & -\bar{F}_f & -\bar{F}_w \\ p\breve{C} & p\breve{G}_f & p\breve{G}_w \\ 0 & pI_s & 0 \\ \breve{C} & \breve{G}_f & \breve{G}_w \end{bmatrix} \\ &= \bar{n} + \operatorname{rank}[\,\bar{F}_f \quad \bar{F}_w\,] \qquad \forall\, \mathbb{R}(p) \geq 0 \\ &\Leftrightarrow (34) \end{aligned}$$

since $\operatorname{rank}\bar{F}_w = q$. ■

References

[1] K. K. Busawon and P. Kabore, "Disturbance attenuation using integral observers," *Int. J. Control*, vol. 74, no. 6, pp. 618–627, 2001.

[2] D. Chu and V. Mehrmann, "Dirturbance decoupled observer design for descriptor systems," *Sys. Control Lett.*, vol. 38, pp. 37–48, 1999.

[3] M. Chilali and P. Gahinet, "H_∞ design with pole placement constraints: an LMI approach," *IEEE Trans. Autom. Control*, vol. 41, no. 3, pp. 358–367, Mar. 1996.

[4] L. Dai, "Impulsive modes and causality in singular systems," *Int. J. Control*, vol. 50, pp. 1267–1281, 1989.

[5] ——, *Singular Control Systems*. Berlin, Germany: Springer-Verlag, 1989.

[6] M. Darouach, M. Zasadzinski, and S. J. Xu, "Full-order observers for linear systems with unknown inputs," *IEEE Trans. Autom. Control*, vol. 39, no. 3, pp. 606–609, Mar. 1994.

[7] M. Darouach, M. Zasadzinski, and M. Hayar, "Reduced-order observer design for descriptor systems with unknown inputs," *IEEE Trans. Autom. Control*, vol. 41, no. 7, pp. 1068–1072, Jul. 1996.

[8] I. A. Hiskens and A. Koeman, "Parameter estimation from power system disturbance measurements," *Proc. Energy Management and Power Delivery*, vol. 2, pp. 667–672, 1998.

[9] M. Hou and P. C. Müller, "Observer for linear descriptor systems with unknown input," *Automatisierungstechnik*, vol. 40, no. 6, pp. 220–227, 1992.

[10] ——, "Disturbance decoupled observer design: A unified viewpoint," *IEEE Trans. Autom. Control*, vol. 39, no. 6, pp. 1338–1341, Jun. 1994.

[11] D. Koenig and S. Mammar, "Design of proportional-integral observer for unknown input descriptor systems," *IEEE Trans. Autom. Control*, vol. 47, no. 12, pp. 2057–2062, Dec. 2002.

[12] H. H. Niemann, J. Stoustrup, B. Shafai, and S. Beale, "LTR design of proportional-integral observers," *Int. J. Robust Nonlinear Control*, vol. 5, pp. 671–693, 1995.

[13] B. Schafai, C. T. Pi, S. Nork, and S. P. Linder, "Proportional integral adaptive observer for parameter and disturbance estimations," in *Proc. IEEE Conf. Decesion and Control*, Las Vegas, NV, 2002, pp. 4694–4699.

[14] H. H. Niemann, J. Stoustrup, and B. Shafai, "Fault detection using PI observers," in *Proc. Control*, Sydney, Australia, 1997, pp. 604–609.

[15] C. R. Rao and S. K. Mitra, *Generalized Inverse of Matrices and its Applications*. New York: Wiley, 1971.

[16] M. Ilic and J. Zaborszky, *Dynamics and Control of Large Electric Power Sytems*: Wiley, 2000.

Delay-Dependent Exponential Stability of Stochastic Systems With Time-Varying Delay, Nonlinearity, and Markovian Switching

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***Abstract*—The problem of delay-dependent stability in the mean square sense for stochastic systems with time-varying delays, Markovian switching and nonlinearities is investigated. Both the slowly time-varying delays and fast time-varying delays are considered. Based on a linear matrix inequality approach, delay-dependent stability criteria are derived by introducing some *relaxation* matrices which can be chosen properly to lead to a less conservative result. Numerical examples are given to illustrate the effectiveness of the method and significant improvement of the estimate of stability limit over some existing results in the literature.**

***Index Terms*—Linear matrix inequality (LMI), Markov chain, stability, Stochastic systems, time delay.**

I. Introduction

Since the introduction of the first model for a jump linear system (JLS) by Krasovskii and Lidskii [8], the JLS has become more popular in the area of control and operations research communities. The JLS is a hybrid system in the form

$$\dot{x}(t) = A(r_t)x(t) \tag{1}$$

where one part of the state $x(t)$ takes value continuously in R^n while another part of the state r_t is a Markov chain taking values in a finite state set $\mathcal{S} = \{1, 2, \ldots, N\}$. One can use the JLS to model different types of dynamical systems subject to abrupt changes in their structure, such as failure prone manufacturing systems, power systems and economics systems etc. In the past decades, the stability and control

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