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Robust fault detection for Uncertain Unknown Inputs LPV system ☆

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ABSTRACT

This paper focuses on robust fault residual generation for Uncertain Unknown Inputs Linear Parameter Varying ($\mathcal{U}-$LPV) systems. Firstly, the problem is addressed in standard LPV systems based on the adaptation of the parity-space approach. The main objective of this approach is to design a scheduled parity matrix according to the scheduling parameters. It results a perfectly decoupled parity matrix face to the system states. Then, the major contribution of this paper relies on the extension to $\mathcal{U}-$LPV systems. Since most of models which represent practical/real systems are subject to parameters variation, unmodeled dynamics and unknown inputs, the approach is clearly justified. The residual synthesis is rewritten in terms of a new optimization problem and solved using Linear Matrix Inequalities (LMIs) techniques. An applicative illustration is proposed and rests on a vehicle lateral dynamic system.

1. Introduction

The issue of Fault Detection (FD) in dynamic systems has received considerable attention in both research and application domains since the last two decades. Among all concrete systems, cars represent a wide field of interest in terms of diagnosis. In effect, it has been noticed the emergence of new safety systems for vehicles such as anti-lock braking (ABS), adaptive cruise control (ACC), electronic stability control (ESC) which are incontrovertible products for modern cars. Those electronic/mechanical systems which are designed to enhance the security in vehicles might lead to extremely dangerous situations in case of failures.

Facing this reality, researchers and engineers are working on solutions to strengthen the reliability by adding supplementary fault detection layers on their equipments. However, most of modern processes are complex systems and the synthesis of such fault detection procedure is extremely difficult to perform. In this field, many model-based fault detection and isolation (FDI) procedures have been carried out by researchers. It can be cited, for instance, analytical redundancy-based methods as proposed in Chow and Willsky (1984) and well recalled in Gertler (1998), some statistical and geometrical methods in Basseville and Nikiforov (1993), Fang, Gertler, Kunwer, Heron, and Barkana (1994), Cao and Gertler (2004), Balas, Bokor, and Szabó (2003), and some observer-based approaches as in Ding (2008), Chen and Patton (1999), Simani, Fantuzzi, and Patton (2002). More recently, researchers focused on optimization-based techniques for fault detection (e.g. $\mathcal{H}_\infty-$based approaches) as proposed in Xinzhi and Shuai (2011), Yueyang and Maiying (2009), Maiying, Ding, Qing-Long, and Qiang (2010) and references therein.

Within the same creative impulse to Apkarian, Gahinet, and Becker (1995), many works have been carried out about Linear Parameter Varying (LPV) systems. Such models have received much attention as long as they can be used to represent nonlinear systems. Moreover, vehicle systems which are strongly nonlinear systems are often modeled as LPV systems. This motivates some researchers from the FD community to develop model-based methods using LPV models (Balas, Bokor, & Szabó, 2002; Balas et al., 2003; Bokor & Balas, 2005; Kwiatkowski, Trimpe, & Werner, 2007; Wang, Wang, Gao, & Wu, 2006). There are two commonly used approaches. First the fault estimation methods where the estimated fault is used as the fault indicating signal. Second, residual generation methods where the residuals are synthesized in order to be robust against modeling errors and unknown inputs.

In this paper, the contribution relies on the design of fault indicators for a general class of Uncertain, Unknown Inputs nonlinear system which can be written as LPV Uncertain systems subject to Unknown Inputs, namely $\mathcal{U}-$LPV systems for simplicity. The approach is based on the synthesis of a residual which provides the information on the faultiness of the system. First, the objectives are expressed for LPV systems. Here, the proposed approach is similar to that presented by the authors in Varrier, Koenig, and Martinez (2012a), but extended from LTI parity-space approach. In this case, the parity matrix is computed depending of the scheduling parameters of the system, to guarantee a perfect decoupling face to system dynamics. The parity-matrix evaluation involves symbolic computation and matrix inversions.

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Furthermore, the major contribution concerns the robustness against uncertainties and parameter variation. The synthesis of the parity matrix is handled by expressing the objectives as an optimization problem. The aim is classical: minimize the residual sensitivity with respect to uncertainties and disturbances and maximize its sensitivity face to faults. The approach of the generalized eigenvalues/eigenvectors – as a generalization of the approach expressed in Ding (2008) – is very hard to implement in this application. Its computation involving symbolic computation is not applicable in this case. Facing this difficulty, the optimization problem is finally rewritten in terms of an LMI optimization problem.

The paper is organized as follows: Section 2 presents the modeling of uncertain LPV systems subject to unknown inputs. Then, Section 3 presents the parity-space based fault detection approach expressed in LPV system. Thus, the main contribution is exposed in Section 4 where the approach is extended to uncertain LPV systems. Finally, an applicative example based on a vehicle lateral dynamic system is handled in Section 5 where the aim is to detect a fault (generally sensor bias) on a lateral acceleration sensor. The performances and improvements of the proposed approach are discussed in the last section.

2. Uncertain LPV modeling

This section settles the system under consideration used for residual synthesis. The system is an Uncertain Unknown Inputs LPV ($\mathcal{U}-$LPV) system. The difference between the scheduling parameters for LPV definition and uncertain parameters is very important and is highlighted in the next subsections.

2.1. System definition

Consider the $\mathcal{U}-$LPV system Σ_Δ defined by

$$\Sigma_\Delta : \begin{cases} x_{k+1} = \overline{A}(\rho_k)x_k + B(\rho_k)u_k + B_d(\rho_k)d_k + B_f(\rho_k)f_k \\ y_k = \overline{C}(\rho_k)x_k + D(\rho_k)u_k + D_d(\rho_k)d_k + D_f(\rho_k)f_k \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the state vector of the system, $y \in \mathbb{R}^m$ the output vector, $u \in \mathbb{R}^l$ the input vector, $d \in \mathbb{R}^l_d$ some unknown inputs and $f \in \mathbb{R}^l_f$ some faults affecting both the state and output vectors of the system. In this representation, matrices $\overline{X}$ stand for uncertain matrices, presented in the following section.

Vector $\rho_k = [\rho_{1k}\ \rho_{2k}\ \cdots\ \rho_{Mk}]$ defines the vector of the scheduling parameters ρ_{ik}, which are known at each sample time.

2.2. Uncertainties modeling

Uncertain matrices are considered in the following additive form:

$$\overline{X}(\rho_k) = X_0(\rho_k) + \sum_{i=1}^{N} \tilde{X}_i(\rho_k)\delta_{ik} \tag{2}$$

where $\tilde{X}_i(\rho_k)$ are known LPV matrices and δ_i are unknown scalars.

Remark 1. In the case of many uncertainties δ_i, the matrix $\tilde{X}$ will be very large, roughly full of zeros. Therefore, a change of basis inspired from a singular value decomposition in matrix $\tilde{X}$ should reduce the complexity of the presented methodology as proposed in Koenig and Mammar (2002).

2.3. Scheduling parameters

In the modeling of the system (Eq. (1)), matrices might depend on the vector $\rho_k = [\rho_{1k}\ \rho_{2k}\ \cdots\ \rho_{Mk}]$. The structure of such matrices is considered as an Affine-LPV form such as

$$Z(\rho_k) = Z_0 + \rho_{1k}Z_1 + \cdots + \rho_{Mk}Z_M \tag{3}$$

In this modeling, each scalar ρ_i is an unknown scalar. However, the main difference between δ_i and ρ_i relies on the fact that scheduling parameters ρ_i can be measured at each sample time while δ_i remain unknown.

2.4. Faults and unknown inputs

In the modeling presented in (1), it has been differentiated unknown inputs d_k from faults f_k. This distinction is particularly relevant since the residual has only to inform on the faultiness of the system. In this case, unknown inputs will not affect the residual status.

On the other hand, it is sometimes required (for fault isolation) that the fault detector should only be sensitive to a certain class of faults $f_{1k} \subset f_k$ and insensitive to another class $f_{2k} \subset f_k$. This requirement can easily be handled be splitting both matrices B_f and D_f in (1) with respect to f_{1k} and f_{2k} as

$$B_f f_k = [B_{f1}\ \ B_{f2}] \begin{bmatrix} f_{1k} \\ f_{2k} \end{bmatrix} \tag{4}$$

and creating the new matrices $B_f^* = B_{f2}$ and $B_d^* = [B_d\ \ B_{f1}]$ (idem for matrices D_f and D_d). Subscripts * stand for the newly defined matrix.

Finally, the residual will be sensitive to faults f_{1k} and insensitive to the others f_{2k}.

2.5. Observability assumption

In the sequel, it is assumed that the pair $(\overline{A}(\rho_k), \overline{C}(\rho_k))$ is always observable for any combination of $\tilde{\delta}$ and ρ. As a consequence, the nominal pair $(A_0(\rho_k), C_0(\rho_k))$ is still observable (case $\tilde{\delta} = \mathbb{O}_{N\times 1}$).

2.6. $\mathcal{U}-$LPV system

Finally, the $\mathcal{U}-$LPV system given in (1) can be rewritten in the following form:

$$\Sigma_\Delta : \begin{cases} x_{k+1} = A_0(\rho_k)x_k + \sum_{i=1}^{N} \tilde{A}_i(\rho_k)\delta_{ik}x_k \\ \qquad + B(\rho_k)u_k + B_d(\rho_k)d_k + B_f(\rho_k)f_k \\ y_k = C_0(\rho_k)x_k + \sum_{i=1}^{N} \tilde{C}_i(\rho_k)\delta_{ik}x_k \\ \qquad + D(\rho_k)u_k + D_d(\rho_k)d_k + D_f(\rho_k)f_k \end{cases} \tag{5}$$

where it is separated the completely known part of the system from the uncertain one.

For the sake of residual generation for system Σ_Δ, it has been considered to decompose the further study in two cases:

- *Case of LPV system*: A solution for residual generation for LPV systems is firstly proposed. The involved methodology is inspired from the classical parity-space approach. The scheduling parameters are taken into account to generate a scheduled residual.
- *Extension to $\mathcal{U}-$LPV systems*: Then, the previously exposed methodology is used to tackle the scheduling parameters. An LMI formulation is finally proposed to handle uncertainties and unknown inputs within residual generation.

Both approaches are finally compared in a practical situation in the last Section 5.

3. Fault detection for LPV systems

In this section, it is proposed to design a fault indicator for LPV systems. The fault detection principle is based on the parity space methodology, inspired from the LTI approach (see Chow & Willsky, 1984; Gertler, 1998), but adapted for LPV ones.

The depicted methodology is necessary for the establishment of the final residual for $\mathcal{U}-$LPV systems presented in Section 4. Comparison between both approaches is presented in the applicative Section 5.

3.1. Principle of the approach

In this section, only the following LPV system Σ_{LPV}, without uncertainties nor unknown inputs is considered:

$$\Sigma_{LPV}: \begin{cases} x_{k+1} = A(\rho_k)x_k + B(\rho_k)u_k \\ y_k \quad = C(\rho_k)x_k + D(\rho_k)u_k \end{cases} \tag{6}$$

where $x \in \mathbb{R}^n$ represents the state of the system, $u \in \mathbb{R}^l$ the controlled input, $y \in \mathbb{R}^m$ its output and ρ_k the scheduling parameters.

Diagnosis based on the parity-space approach is generated via linear combinations of measurements (sensors) and applied inputs (actuators) taken over a finite window. By making use of known data, it is generated analytical relationships that hold in the absence of failures. A fault is so detected when equations are no longer verified.

The key of the approach is to express the output and its time shifted over a horizon s as

$$\underbrace{\begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+s) \end{bmatrix}}_{Y_s} = \underbrace{\begin{bmatrix} C(\rho_k) \\ C(\rho_{k+1})A(\rho_k) \\ \vdots \\ C(\rho_{k+s})\prod_{i=1}^{s-1} A(\rho_{k+s-i-1}) \end{bmatrix}}_{H_{os}(\rho_k)} x(k) + H_{us}(\rho_k) \underbrace{\begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+s) \end{bmatrix}}_{U_s} \tag{7}$$

with

$$H_{us}(\rho_k) = \begin{bmatrix} D(\rho_k) & 0 & \cdots & \cdots & \cdots & 0 \\ C(\rho_{k+1})B(\rho_k) & \cdots & \cdots & 0 & \cdots & 0 \\ C(\rho_{k+2})A(\rho_{k+1})B(\rho_k) & C(\rho_{k+1})B(\rho_k) & \cdots & D(\rho_k) & \ddots & \vdots \\ \vdots & \ddots & \cdots & \ddots & \ddots & 0 \\ C(\rho_{k+s})\prod_{i=1}^{s-1} A(\rho_{k+s-i})B(\rho_k) & C(\rho_{k+s-1})\prod_{i=2}^{s-1} A(\rho_{k+s-i})B(\rho_k) & \cdots & \cdots & C(\rho_{k+1})B(\rho_k) & D(\rho_k) \end{bmatrix} \tag{8}$$

Remark 2. The observability matrix $H_{os}(\rho_k)$ is expressed with products of matrices A. However, it is known that high powers of matrix A are generally numerically unstable. Thus, as the approach is based on this matrix, the result could be numerically unstable. So, in the case of a high horizon s, the interested author should use another construction of the observability matrix as discussed in Varga (1981).

In Eq. (7), Y_s is built from sensor measurements and U_s results of the applied inputs. Those data are perfectly known. However, the state $x(k)$ is – a priori – unknown.

The parity-space methodology manages to make the term $H_{os}(\rho_k)x(k)$ vanish. Left multiplying Eq. (7) by the so-called parity matrix $W(\rho_k)$, the residual $r(k)$ is finally given by

$$r(k) = W(\rho_k)^T(Y_s - H_{us}(\rho_k)U_s) \tag{9}$$

where the parity matrix $W(\rho_k)$ is selected from the parity space $\mathcal{P}$ defined by

$$\mathcal{P} \triangleq \{W(\rho_k) \text{ s.t. } W(\rho_k)^T H_{os}(\rho_k) = 0\}$$

Thus in absence of failures, the residual is null.

Moreover, for fault isolation, the usual DOS (dedicated observer scheme) or GOS (generalized observer scheme) schemes (Frank, 1990) can be applied.

3.2. Parity matrix synthesis

Now, the main objective consists in finding the parity matrix $W(\rho_k)$ which has to be orthogonal to the matrix $H_{os}(\rho_k)$, i.e. $W(\rho_k)\cdot H_{os}(\rho_k) = 0$.

First, the matrix $H_{os}(\rho_k)$ can always be split into two sub-matrices as follows:

$$H_{os}(\rho_k) = \begin{bmatrix} H_{os1}(\rho_k) \\ H_{os2}(\rho_k) \end{bmatrix} \tag{10}$$

where the matrix $H_{os1}(\rho_k)$ is regular and so invertible.

Remark 3. By assumption, the system (6) is observable and so the observability matrix $\mathcal{O}(A(\rho_k), C(\rho_k))$ is full rank. As a consequence, for a horizon s chosen $s \geq E(n/m)$ (where $E(\cdot)$ represents the superior integer part), it is always possible to choose the matrix $H_{os1}(\rho_k)$ as the regular part of the observability matrix of the nominal system, i.e. $H_{os1}(\rho_k) = reg(\mathcal{O}(A(\rho_k), C(\rho_k)))$. So for a SISO system, s_1 can be chosen as $s_1 = n$.

Thus, by defining $W(\rho_k)^T = [W_1(\rho_k)^T \;\; W_2(\rho_k)^T]^T$, the null equality $W(\rho_k)^T \cdot H_{os}(\rho_k) = 0$ can be expressed as

$$W(\rho_k)^T H_{os}(\rho_k) = 0 \Leftrightarrow [W_1(\rho_k)^T \;\; W_2(\rho_k)^T] \begin{bmatrix} H_{os1}(\rho_k) \\ H_{os2}(\rho_k) \end{bmatrix} = 0$$

$$\Leftrightarrow W_1(\rho_k)^T = -W_2(\rho_k)^T H_{os2}(\rho_k) H_{os1}(\rho_k)^{-1} \tag{11}$$

Finally, the matrix $W(\rho_k)^T$ can be rewritten as

$$\begin{aligned} W(\rho_k)^T &= [W_1(\rho_k)^T \;\; W_2(\rho_k)^T]^T \\ &= [-W_2(\rho_k)^T H_{os2}(\rho_k) H_{os1}(\rho_k)^{-1} \;\; W_2(\rho_k)^T] \\ &= W_2(\rho_k)^T \underbrace{[-H_{os2}(\rho_k)H_{os1}(\rho_k)^{-1} \;\; \mathbb{I}_q]}_{P(\rho_k)} \end{aligned}$$

$$W(\rho_k)^T = W_2(\rho_k)^T P(\rho_k) \tag{12}$$

According to those algebraic manipulations, the parity matrix $W(\rho_k)$ is built from the knowledge of matrices $A(\rho_k)$ and $C(\rho_k)$. The left-hand side matrix $W_2(\rho_k)$ stands as an extra degree of freedom. It can be chosen unitary for the purpose of this section.

Nevertheless, the matrix W_2 can be used in order to increase the robustness of the system face to uncertainties, as used in the next section.

3.3. Choice of the horizon s

According to the depicted methodology, the horizon s in (7) affects the definition of the residual $r(k)$. Indeed, thanks to Eq. (10) and Remark 3, the horizon has to be chosen at least larger than the size of the observability matrix $\mathcal{O}(A(\rho_k), C(\rho_k))$. The leading $H_{os1}(\rho_k)$ matrix will be invertible. Moreover, the matrix $H_{os2}(\rho_k)$ defines supplementary rows in the H_{os} matrix. Those redundant rows are constructing new residuals in the vector $r(k)$.

To sum up, it is required that the horizon s guarantees the definition of the observability matrix, so using the definition in Eq. (6), $s \geq n/m$, plus the number of desired residuals. For the sake of this section, where one residual is required, the horizon can be chosen as recalled in Varrier et al. (2012a):

$$s = E\left(\frac{n}{m}\right) \tag{13}$$

where $E(\cdot)$ represents the superior integer part.

To conclude, the choice of the horizon for residual generation has to be done respecting Eq. (13), but might be increased to enhance redundant information within the residual.

On the other hand, the residual synthesis involves inversion of a symbolic matrix $H_{os1}(\rho_k)$ according to the scheduling parameters ρ_k see Eqs. (9) and (12). This symbolic inversion is not restrictive as it can easily be handled via symbolic software.

4. Fault detection for $\mathcal{U}-$LPV systems

Since a fault detection procedure has been proposed for LPV system, it is now extended to $\mathcal{U}-$LPV ones. Direct extension from the previous approach eliminating all the terms subject to uncertainties and unknown inputs is not available. The further subsection justifies this statement. So, an optimization procedure based on an LMI formulation of the problem is proposed. The aim is to synthesize a residual mainly sensitive to the faults while being non-receptive to uncertainties nor unknown inputs whatever the scheduling parameters are.

4.1. Extension of the LPV approach

Now, the system under consideration is the $\mathcal{U}-$LPV system (5). Similar to Section 3.1, expressing the output y_k along the horizon s yields

$$\begin{aligned} Y_s(k)-H_{us}U_s(k) &= H_{os}x(k)+\sum_i(\zeta_k(i)\tilde{H}_{os,i})x(k) \\ &+\sum_i(\zeta_k(i)\tilde{H}_{us,i})U_s(k)+H_{ds}U_{ds}(k) \\ &+\sum_i(\zeta_k(i)\tilde{H}_{ds,i})U_s(k)+H_{fs}F_s(k) \end{aligned} \tag{14}$$

where ζ_k is constructed with δ_k, the powers of its elements and multiple inner products in the form

$$\tilde{\zeta}_k^T = [\delta_{1k}\ \cdots\ \delta_{Nk}\ \delta_{1k}^2\ \delta_{1k}\delta_{2k}\ \cdots\ \delta_{1k}^p\ \delta_{1k}^{p-1}\delta_{2k}\ \cdots\ \delta_{Nk}^{s+1}]^T$$

where $\delta_{ik}^q = \underbrace{\delta_{ik}\delta_{ik}\cdots\delta_{ik}}_{\text{q products}}$.

The (trivial) classical parity space approach would consist in finding a scheduled parity matrix $W(\rho_k)$ ensuring $W(\rho_k)^T H_{os}(\rho_k)=0$. It leads to a residual sensitive to the unknown inputs d_k, the uncertainties δ_{ik} and to the faults f_k. This conspicuous solution is not acceptable for our problem.

Otherwise, it may sometimes be found a parity matrix $W(\rho_k)$ satisfying

$$W(\rho_k)^T[H_{os}(\rho_k)\ \tilde{H}_{os}(\rho_k)\ \tilde{H}_{us}(\rho_k)\ H_{ds}(\rho_k)\ \tilde{H}_{ds}(\rho_k)]=0 \tag{15}$$

where $\tilde{H}_{os}(\rho_k)=[\tilde{H}_{os,1}(\rho_k)\ \cdots\ \tilde{H}_{os,z}(\rho_k)]$.

First, note that the existence of such a matrix is not guaranteed since the rowrank of the matrix $[H_{os}(\rho_k)\ \tilde{H}_{os}(\rho_k)\ \tilde{H}_{us}(\rho_k)\ H_{ds}(\rho_k)\ \tilde{H}_{ds}(\rho_k)]$ is not necessarily degenerated. Nevertheless, in the case of existence, the horizon should be high, leading to hard computations (numerical troubles due to high powers of matrix A as exposed in Remark 2) and a long fault detection time, as the residual will be sensitive to faults during all the horizon s.

As a consequence, the perfect decoupling is not suitable for on-line computation. A non-perfect parity matrix $W(\rho_k)$ will be sought via an optimization procedure, as introduced in the problem formulation (17).

4.2. Optimization procedure

The aim of the problem is to find a residual $r(k)$ only designed from known data as

$$r(k) = W(\rho_k)^T(Y_s(k)-H_{us}(\rho_k)U_s(k)) \tag{16}$$

which has to be sensitive to faults f_k and insensitive to uncertainties δ_i and to unknown input d_k. Those objectives can be expressed in the following optimization problem in the variable $W(\rho_k)$:

$$\text{find } W(\rho_k) \text{ s.t. : } \begin{cases} W(\rho_k)^T H_{os}=0 \\ \max\limits_W \|W(\rho_k)^T H_{fs}(\rho_k)\|^2 \\ \min\limits_W \|W(\rho_k)^T \tilde{H}_{os}(\rho_k)\|^2 \\ \min\limits_W \|W(\rho_k)^T \tilde{H}_{us}(\rho_k)\|^2 \\ \min\limits_W \|W(\rho_k)^T H_{ds}(\rho_k)\|^2 \\ \min\limits_W \|W(\rho_k)^T \tilde{H}_{ds}(\rho_k)\|^2 \end{cases} \tag{17}$$

This problem (17) can be written as a constrained optimization problem (Ding, 2008; Varrier, Koenig, & Martinez, 2012b), as follows:

$$\mathcal{P}_1 : \begin{cases} W(\rho_k)^T H_{os}=0 \\ \min\limits_W \dfrac{\|W(\rho_k)^T G(\rho_k)\|^2}{\|W(\rho_k)^T H_{fs}(\rho_k)\|^2} \end{cases} \tag{18}$$

where $G(\rho_k)=[\tilde{H}_{os}(\rho_k)\ \tilde{H}_{us}(\rho_k)\ H_{ds}(\rho_k)\ \tilde{H}_{ds}(\rho_k)]$.

Following the methodology proposed in Section 3.2, this constrained optimization problem can be turned into a classical unconstrained one.

The constraint $W(\rho_k)^T H_{os}=0$ is guaranteed by considering $W(\rho_k)$ as in (12). Therefore, the optimization problem (18) becomes an unconstrained optimization problem in the variable $W_2(\rho_k)$ as

$$\mathcal{P}_1 : \min_{W_2} \frac{\|W_2(\rho_k)^T P(\rho_k) G(\rho_k)\|^2}{\|W_2(\rho_k)^T P(\rho_k) H_{fs}(\rho_k)\|^2} \tag{19}$$

which is equivalent to

$$\begin{aligned} \mathcal{P}_1 : &\min_{W_2} W_2(\rho_k)^T \frac{\overbrace{P(\rho_k)G(\rho_k)G(\rho_k)^T P(\rho_k)^T}^{\Gamma_1(\rho_k)} W_2(\rho_k)}{W_2(\rho_k)^T \underbrace{P(\rho_k)H_{fs}(\rho_k)H_{fs}(\rho_k)^T P(\rho_k)^T}_{\Gamma_2(\rho_k)} W_2(\rho_k)} \\ &= \min_{W_2} \frac{W_2(\rho_k)^T \Gamma_1(\rho_k) W_2(\rho_k)}{W_2(\rho_k)^T \Gamma_2(\rho_k) W_2(\rho_k)} \end{aligned} \tag{20}$$

where $\Gamma_1(\rho_k)$ and $\Gamma_2(\rho_k)$ are symmetric matrices.

Now, problem $\mathcal{P}_1$ stands as an unconstrained quadratic optimization problem in the variable $W_2(\rho_k)$, but scheduled by parameter ρ_k. In the next sections, it is proposed a theorem that symbolically solves (20) depending of the scheduling parameters. Thus a new polytopic/LMI formulation is studied to avoid symbolic computations.

4.3. Resolution by eigenvalue assignment

The following theorem and its associated proof give one solution to problem $\mathcal{P}_1$.

Theorem 1 (*See Ding, 2008 for the proof*). *Given an optimization problem of the form*

$$\gamma^* = \min_X \frac{X^T A X}{X^T B X} \tag{21}$$

where A and B are symmetric matrices, the minimum γ corresponding to the criterion given in (21) is reached by X^* such that

$$X^* = \vartheta_{\lambda_q(A,B)} \tag{22}$$

where $\lambda_q(A,B)$ stands for the lowest generalized eigenvalue of the pair (A,B), and $\vartheta_{\lambda_q(A,B)}$ its associated eigenvector. The minimum γ^* is given by $\gamma^* = \lambda_q(A,B)$.

Thanks to Theorem 1, problem $\mathcal{P}_1$ (20) can be solved by

$$\mathcal{P}_1 : W_2(\rho_k)^T = \vartheta_{\lambda_q(\Gamma_1(\rho_k),\Gamma_2(\rho_k))}(\rho_k) \tag{23}$$

Then, it is easy to recover the parity vector $W(\rho_k)$ thanks to Eq. (12).

This theorem clearly solves problem $\mathcal{P}_1$. It has to be pointed out that its resolution involves to compute generalized eigenvalues and eigenvectors, depending on the parameter ρ_k. As each ρ_i belongs in an infinite dimensional set ($\rho_i \in [\rho_{min} \;\; \rho_{max}]$), the resolution of infinite dimension is impossible. On the other hand, in the case of smart and small dimension systems, where it has been considered few uncertainties, this computation can be handled symbolically. However, most of systems are complex, the resolution involves very hard computation and consequently hard implementation.

Facing this reality, this symbolic resolution has been given up making use of a polytopic approach. Then the optimization problem is rewritten in a new polytopic form, where each vertex represents constant sub-optimization problems. In this way, instead of considering the scheduling parameter within the resolution, computations are made at each vertex of the polytope, synthesizing as many parity vectors W_i as polytope vertices. Thus the parity vector $W(\rho_k)$ is built from the linear combination of each W_i.

4.4. LMI optimization formulation

The problem $\mathcal{P}_1$ can be rewritten as the problem $\mathcal{P}_2$ (Ding, 2008):

$$\mathcal{P}_2 : \left\{ \gamma_2(\rho_k) = \min_{W_2} W_2(\rho_k)^T(\Gamma_1(\rho_k) - \kappa^2\Gamma_2(\rho_k))W_2(\rho_k) \right. \tag{24}$$

In effect, the aim is to find a vector $W_2(\rho_k)$ which minimizes the sensibility of uncertainties and disturbances – here represented by matrix $\Gamma_1(\rho_k)$ – and maximizes the effect of faults, represented by matrix $\Gamma_2(\rho_k)$.

Remark 4. The problems $\mathcal{P}_1$ and $\mathcal{P}_2$ are not strictly equals. They can be considered as strictly equals if κ is chosen as the lowest eigenvalue of the pair (Γ_1, Γ_2). However, in our case, κ is just chosen as an extra degree of freedom to fulfill the further constraints. Nevertheless, even if both problems are not strictly equals, they traduce the same objective: minimize the effect of the disturbances (Γ_1) and maximize the effect of the fault (Γ_2).

To better understand the process formulation, the problem is firstly addressed for LTI systems (Section 4.4.1) and secondly for LPV systems (Section 4.5).

4.4.1. Case of LTI systems

Consider the optimization problem for classical LTI matrices $\Gamma_{i=1,2}$. The aim of the problem is to find a vector W_2 s.t.

$$\gamma = \min_{W_2} W_2^T \underbrace{(\Gamma_1 - \kappa^2\Gamma_2)}_{\Gamma_\kappa} W_2 \tag{25}$$

It can be rewritten as a BMI problem

$$\begin{cases} \underset{W_2}{\text{minimize}} & \gamma \\ \text{subject to} & W_2^T \Gamma_\kappa W_2 \prec \gamma \end{cases} \tag{26}$$

which can be expressed as an LMI thanks to the Schur Lemma on Eq. (26):

$$\begin{cases} \underset{W_2}{\text{minimize}} & \gamma \\ \text{subject to} & \begin{bmatrix} \Gamma_\kappa^{-1} & W_2 \\ W_2^T & \gamma \end{bmatrix} \succ 0 \end{cases} \tag{27}$$

where Γ_κ has to guarantee $\Gamma_\kappa \succ 0$.

Remark 5. The constraint $\Gamma_\kappa \succ 0$ has to be guaranteed due to the Schur Lemma. This condition is guaranteed by choosing properly the scalar κ. For the sequel, it is considered that κ is properly defined so the constraint is no more a part of the problem.

From the problem given in (26), a trivial solution consists in choosing W_2 as a null matrix. This solution is not allowable for our problem. So an additional constraint is added in order to tackle this problem

$$W_2 \neq \mathbb{O}_{1\times m\cdot s} \tag{28}$$

This constraint cannot be directly implemented as an LMI since the vector W_2 is not a symmetric square matrix. As a consequence, it is considered a new diagonal matrix Y, defined by

$$Y = \text{diag}(W_2) \tag{29}$$

from which it can be derived the following constraint:

$$\text{tr}(|Y|) \succ \varepsilon, \quad \varepsilon > 0 \tag{30}$$

Finally, the optimization problem can be rewritten in terms of LMI optimization as

$$\begin{cases} \underset{W_2}{\text{minimize}} & \gamma \\ \text{subject to} & \begin{cases} \begin{bmatrix} \Gamma_\kappa^{-1} & W_2 \\ W_2^T & \gamma \end{bmatrix} \succ 0 \\ \text{tr}(|Y|) - \varepsilon \succ 0 \qquad \varepsilon > 0 \end{cases} \end{cases} \tag{31}$$

This LMI formulation allows us to solve the problem $\mathcal{P}_2$ for LTI matrices. The next section presents a way to extend the results to LPV systems.

4.5. Extension to LPV systems

Applying directly the results of the proposed approach for LPV systems is not allowable as it involves infinite LMI resolution among all the parameter set. However, the matrix $\Gamma_\kappa(\rho_k)$ (built from $\Gamma_1(\rho_k)$ and $\Gamma_2(\rho_k)$) can be rewritten in a polytopic form as

$$\Gamma_\kappa(\rho_k) = \sum_{i=1}^{q} \alpha_i(\rho_k)\Gamma_{ki} \tag{32}$$

The aim of working in the polytopic framework relies on the fact that the study can be applied to each subsystem at each vertex of the polytope.

However, by extension of (31), it has to be computed the inverse of the matrix $\Gamma_\kappa(\rho_k)$. So considering directly the polytopic formulation of the matrix $\Gamma_k(\rho_k)$ is not useful since it should not preserve the linearity in the parameters α_i. Nevertheless, the interest relies in the polytopic formulation of its inverse $\Gamma_k^{-1}(\rho_k)$ which can be expressed as

$$\Gamma_\kappa^{-1}(\rho_k) = \alpha_0(\rho_k) \sum_{i=1}^{r} \tilde{\alpha}_i(\rho_k)\tilde{\Gamma}_{ri} \tag{33}$$

where the term $\alpha_0(k)$ stands for the inverse of the determinant of matrix $\Gamma_\kappa(\rho_k)$. Moreover, as the matrix $\Gamma_\kappa(\rho_k)$ has to be positive definite for all parameters ρ_k, its associated determinant is also positive.

Remark 6. According to the polytopic formulation of the system, each $\tilde{\alpha}_i$ is positive. So, positive definiteness of the product $\tilde{\alpha}_i \tilde{\Gamma}_{ri}$ is only affected by $\tilde{\Gamma}_{ri}$.

The LMI optimization problem can be written as

$$\begin{cases} \underset{W_2}{\text{minimize}} & \gamma \\ \text{subject to} & \gamma - W_2^T \Gamma_\kappa(\rho_k) W_2 \succ 0 \end{cases} \tag{34}$$

where the constraint can be developed as

$$\begin{aligned} &\gamma - W_2^T \Gamma_\kappa(\rho_k) W_2 \succ 0 \\ &\Leftrightarrow \gamma - W_2^T \left[\alpha_0(\rho_k) \sum_{i=1}^{r} \tilde{\alpha}_i(\rho_k) \tilde{\Gamma}_{ri} \right]^{-1} W_2 \succ 0 \\ &\Leftrightarrow \gamma - \frac{1}{\alpha_0(\rho_k)} W_2^T \left[\sum_{i=1}^{r} \tilde{\alpha}_i(\rho_k) \tilde{\Gamma}_{ri} \right]^{-1} W_2 \succ 0^1 \\ &\Leftrightarrow \underbrace{\gamma \alpha_0(\rho_k)}_{\gamma'(\rho_k)} - W_2^T \left[\sum_{i=1}^{r} \tilde{\alpha}_i(\rho_k) \tilde{\Gamma}_{ri} \right]^{-1} W_2 \succ 0 \\ &\Leftrightarrow \gamma'(\rho_k) - W_2^T \left[\sum_{i=1}^{r} \tilde{\alpha}_i(\rho_k) \tilde{\Gamma}_{ri} \right]^{-1} W_2 \succ 0 \end{aligned} \tag{35}$$

Applying the Schur complement on (35) yields the LMI constraint:

$$\begin{bmatrix} \sum_{i=1}^{r} \tilde{\alpha}_i(\rho_k) \tilde{\Gamma}_{ri} & W_2 \\ W_2^T & \gamma' \end{bmatrix} \succ 0 \tag{36}$$

Choosing the vector W_2 in the polytopic form

$$W_2 = \sum_{i=1}^{r} \tilde{\alpha}_i(\rho_k) W_i \tag{37}$$

and the same structure for $\gamma'(\rho_k)$ allows us to solve r LMIs:

$$\sum_{i=1}^{r} \tilde{\alpha}_i(\rho_k) \begin{bmatrix} \tilde{\Gamma}_{ri} & W_{2i} \\ W_{2i}^T & \gamma' \end{bmatrix} \succ 0 \tag{38}$$

which is guaranteed by

$$\begin{bmatrix} \tilde{\Gamma}_{ri} & W_{2i} \\ W_{2i}^T & \gamma' \end{bmatrix} \succ 0 \quad \forall i \in [\![1:r]\!] \tag{39}$$

Adapting Eq. (31) with the LPV constraint (39), the problem $\mathcal{P}_2$ is obtained by the following optimization problem:

$$\begin{cases} \underset{W_2}{\text{minimize}} & \gamma' \\ \text{subject to} & \begin{cases} \begin{bmatrix} \tilde{\Gamma}_{ri} & W_{2i} \\ W_{2i}^T & \gamma' \end{bmatrix} \succ 0 & \forall i \in [\![1:r]\!] \\ \text{tr}(|Y_i|) - \varepsilon \succ 0 \quad \varepsilon > 0 \end{cases} \end{cases} \tag{40}$$

where each Y_i is defined by extension of the LTI case as $Y_i = \text{diag}(X_i)$.

Remark 7. In this formulation, the fact that each matrix $\tilde{\Gamma}_{ri}$ – defining the polytopic formulation of the problem – has to be positive definite has been omitted. In effect, the initial matrix $\Gamma_\kappa(\rho_k)$ has been constructed in order to be positive definite. However, nothing guarantees that each sub-matrices $\tilde{\Gamma}_{ri}$ is positive definite (as they can represent systems with no physical meaning), except for ones which really belong to the parameter definition.

[1] $\alpha_0(\rho_k) \neq 0$ as $\Gamma_\kappa(\rho_k)$ is positive definite.

Case of non-positive definite matrices: In agreement with Remark 7, some matrices written $\tilde{\Gamma}_{ri}^-$ might not be positive definite, and can be decomposed via a singular value decomposition as

$$\tilde{\Gamma}_{ri}^- = V_n \cdot \underbrace{\begin{bmatrix} D_{n1} > 0 & \mathbb{O} \\ \mathbb{O} & D_{n2} < 0 \end{bmatrix}}_{D_n} \cdot V_n^T \tag{41}$$

where V_n corresponds to the eigenvectors of $\tilde{\Gamma}_{ri}^-$ and D_n the matrix of its eigenvalues. Matrix D_{n1} represents the positive eigenvalues of matrix D_n while D_{n2} the negative ones.

In order to avoid the problem of non-positive definiteness, the sub-matrix D_{n2} is replaced by a matrix D_{n2}^+ with positive values, large enough in order not to affect the result of the optimization. In effect, it is known that the optimum rise to the lowest eigenvalue of the system (see Theorem 1).

Remark 8. The best solution is given by the eigenvector associated to the lowest eigenvalue of the inverse of $\tilde{\Gamma}_{ri}^-$. The eigenvalues D_{n2}^+ are said to be "large enough" in order not to affect the result of the optimization. Large enough refers to "at least larger than the other ones". In this case, the optimum reached by the optimization procedure will not be affected.

It is finally considered the matrix $\check{\Gamma}_{ri}$ instead of $\tilde{\Gamma}_{ri}^-$ for the optimization process defined by

$$\check{\Gamma}_{ri} = V_n \cdot \underbrace{\begin{bmatrix} D_{n1} & \mathbb{O} \\ \mathbb{O} & D_{n2}^+ \end{bmatrix}}_{D_n} \cdot V_n^T \tag{42}$$

Let us just note that the transformation from $\tilde{\Gamma}_{ri}^-$ to $\check{\Gamma}_{ri}$ does not restrict the problem solutions. It is just a way to avoid unavailable solutions.

4.6. Interest of small time varying parameters

In the past section, it has been highlight the interest of the polytopic formulation of the matrix $\Gamma_k^{-1}(\rho_k)$. The main drawback of such approach is that it leads to a more conservative solution set and foremost a large number of subsystems $\tilde{\Gamma}_{ri}$, 2^N where N stands for the number of individual scheduling parameters defining the matrix $\Gamma_\kappa^{-1}(\rho_k)$. The following presents tricks to reduce the polytope size and so the complexity of the solution.

4.6.1. Taylor approximation

It is known the following equality as the Taylor development of a function f:

$$f(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \tag{43}$$

where $f^{(n)}$ denotes the nth derivative of a function (43) at the point a.

According to this property, the expression of the scheduling parameters and their time shift can be expressed as

$$\rho(k+\tau) = \rho(k) + \tau \cdot \underbrace{\rho'(k)}_{\leq M_\rho} + \sum_{n=2}^{+\infty} \frac{\rho^{(n)}(k)}{n!} (\tau)^n \tag{44}$$

$$\rho(k+\tau) \simeq \rho(k) + \tau M_\rho \tag{45}$$

where M_ρ represents the maximum variation of parameter ρ.

This simplification (if it can be assumed) is very useful as it allows us to simplify the problem. If the scheduling parameters can be considered constant within the time horizon s, it will lead to a residual constructed with only the knowledge of ρ_k instead of $\rho_{k-s:k}$. Finally, the inner products of time shifts are only

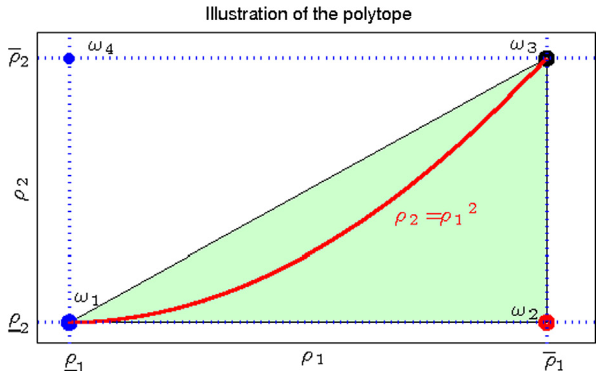

Fig. 1. Illustration of the polytope reduction.

constructed with the knowledge of the scheduling parameter ρ_k. Thanks to this simplification, the further polytope reduction can be adopted.

4.6.2. Polytope reduction

In the case of small time varying parameters, the scheduling parameters can be considered as constant within the time window defined by the horizon.

Thus, it will only remain in the matrix Γ_k^{-1} the term ρ_k and its powers in the form $\Gamma_\kappa^{-1}(\rho_k, \rho_k^2, \ldots, \rho_k^N)$. The classical polytopic modeling will need to define $P = 2^r$ vertices of the polytope.

However, the relation between each parameter is not taken into account. For instance, in the case of two scheduling parameters (as illustrated in Fig. 1), the vertex $\omega_4 = (\underline{\rho}_1, \overline{\rho}_2)$ is not required since the three other vertices are sufficient to characterize the parameter definition.

This reductive approach can be generalized to polytopes of dimension N since the minimal value of the scheduling parameter is null (as presented in Robert, 2007):

Property 1. *A vertex $\omega_j = (\nu_1, \nu_2, \ldots, \nu_N)$ – where each coordinate ν_i is defined as $\nu_i = \{\rho_{min} = 0, \rho_{max}\}$ – is an admissible vertex if its coordinates verify the following property*:

$$\nu_{n+1} \le \rho_{max} \nu_n \tag{46}$$

Therefore, as a simplification, only $N+1$ vertices are finally useful instead of 2^N.

Thanks to those presented tricks, the computations are simplified which allow for implementation on a real vehicle, as presented in the following section.

5. Application on a vehicle lateral dynamics system

The previous depicted theory has been applied to a vehicle lateral dynamic system. Some experimental data have been taken from a real vehicle "Renault Scenic" (Fig. 2). Those data have been provided by the French laboratory MIPS[2] (Modélisation, Intelligence, Processus et Systèmes), a partner in the framework of the French ANR project INOVE.

5.1. Modeling of the system

The aim of this applicative study is to detect a fault on a lateral acceleration sensor. The system under consideration is the whole lateral dynamic system illustrated in Fig. 3.

[2] http://www.mips.uha.fr/.

Fig. 2. Photo of the Renault Scenic used.

The modeling of the system rests on the bicycle model as presented in Poussot-Vassal (2008), Mammar and Koenig (2002), and Ackermann and Bunte (1997), where the dynamic is given by

$$\begin{bmatrix} \dot\beta(t) \\ \dot r(t) \end{bmatrix} = \begin{bmatrix} -\frac{c_{\alpha V} + c_{\alpha H}}{mv(t)} & \frac{l_H c_{\alpha H} - l_V c_{\alpha V}}{mv(t)^2} - 1 \\ \frac{l_H c_{\alpha H} - l_V c_{\alpha V}}{I_z} & \frac{l_V^2 c_{\alpha V} + l_H^2 c_{\alpha H}}{I_z v(t)} \end{bmatrix} \begin{bmatrix} \beta(t) \\ r(t) \end{bmatrix} + \begin{bmatrix} \frac{c_{\alpha V}}{mv(t)} \\ \frac{l_V c_{\alpha V}}{I_z} \end{bmatrix} u_L(t)$$

$$\begin{bmatrix} a_y(t) \\ r(t) \end{bmatrix} = \begin{bmatrix} -\frac{c_{\alpha V} + c_{\alpha H}}{m} & \frac{l_H c_{\alpha H} - l_V c_{\alpha V}}{mv(t)^2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta(t) \\ r(t) \end{bmatrix} + \begin{bmatrix} \frac{c_{\alpha V}}{m} \\ 0 \end{bmatrix} u_L(t) \tag{47}$$

where β denotes the side slip angle, r the yaw rate, a_y the lateral acceleration, u_L the relative steering wheel angle and $v(t)$ the speed of the vehicle. This model represents the nominal behavior of the system.

The numerical values and the description of the parameters are given in the following table:

Variable	Value	Unit	Comments
g	9.80665	m s^{-2}	Gravity acceleration constant
m	1621	kg	Vehicle total mass
l_V	1.15	m	Distance from C.G. to front axle
l_H	1.38	m	Distance from C.G. to rear axle
I_z	1975	kg m^2	Moment of inertia about the z-axis
$c_{\alpha V}$	57 117	N rad^{-1}	Front axle tire cornering stiffness
$c_{\alpha H}$	81 396	N rad^{-1}	Rear axle tire cornering stiffness
v		m s^{-1}	Vehicle longitudinal velocity
β		rad	Vehicle side slip angle
$\dot\psi(t)$		rad s^{-1}	Vehicle raw rate
a_y		m s^{-2}	Vehicle lateral acceleration
u_L		rad	Vehicle steering angle

As presented in van den Hof, Tóth, and Heuberger (2010), several discretization techniques for LPV state space models are available. Here the rectangular discretization is chosen as it provides a low computational load, and preserves the linearity in the parameters. The discrete matrices are given by

$$A_0(\rho(k)) = \mathbb{I}_n + T_d A(\rho(kT_d))$$

$$B_0(\rho(k)) = T_d B(\rho(kT_d))$$

$$C_0(\rho(k)) = C(\rho(kT_d))$$

$$D_0(\rho(k)) = D(\rho(kT_d))$$

where T_d is the sampling period. Thus, the nominal discrete LPV matrices (as in (5)) corresponding to the state space model are given in the following equation:

$$A_0(\rho_k) = \begin{bmatrix} 1 & -T_d \\ T_d \frac{l_H c_{\alpha H} - l_V c_{\alpha V}}{I_z} & 1 \end{bmatrix} + \underbrace{\frac{1}{v(kT_d)}}_{\rho_{1k}} \begin{bmatrix} -T_d \frac{c_{\alpha V} + c_{\alpha H}}{m} & 0 \\ 0 & -T_d \frac{l_v^2 c_{\alpha V} + l_H^2 c_{\alpha H}}{I_z} \end{bmatrix}$$

$$+ \underbrace{\frac{1}{v(kT_d)^2}}_{\rho_{2k}} \begin{bmatrix} 0 & T_d \frac{l_H c_{\alpha H} - l_v c_{\alpha V}}{m} \\ 0 & 0 \end{bmatrix} \tag{48a}$$

$$B_0(\rho_k) = \begin{bmatrix} 0 \\ T_d \frac{l_V c_{\alpha V}}{I_z} \end{bmatrix} + \rho_{1k} \begin{bmatrix} T_d \frac{c_{\alpha V}}{m} \\ 0 \end{bmatrix} \tag{48b}$$

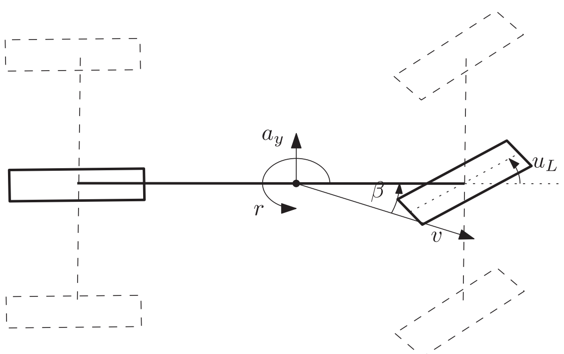

Fig. 3. Illustration of the vehicle lateral dynamics.

$$C_0(\rho_k) = \begin{bmatrix} -\frac{c_{\alpha V} + c_{\alpha H}}{m} & 0 \\ 0 & 1 \end{bmatrix} + \rho_{1k} \begin{bmatrix} 0 & \frac{l_H c_{\alpha H} - l_V c_{\alpha V}}{m} \\ 0 & 0 \end{bmatrix} \tag{48c}$$

$$D_0(\rho_k) = \begin{bmatrix} \frac{c_{\alpha V}}{m} \\ 0 \end{bmatrix} \tag{48d}$$

where there are two scheduling parameters $\rho_{1k} = 1/v(kT_d)$ and $\rho_{2k} = 1/v(kT_d)^2$. However, the dependency between parameters ρ_1 and ρ_2 is clear $\rho_2 = \rho_1^2$. This relation is taken for further computations. Only one scheduling parameter ρ_{1k} is finally considered.

Note that these matrices $A_0(\rho_k)$, $B_0(\rho_k)$, $C_0(\rho_k)$ and $D_0(\rho_k)$ represent the nominal model of the system.

5.2. Data selection

MIPS laboratory provided data on the speed of the vehicle, steering wheel angle and lateral acceleration. The steering wheel angle and the vehicle longitudinal speed are respectively presented in Fig. 4a and b. A comparison between the data from MIPS and the corresponding bicycle model outputs is illustrated in Fig. 4c.

In can be observed that the bicycle model fits the real vehicle dynamics. However, some differences can be noticed especially due to the neglected dynamics, the modeling approximations and inaccurate parameters. This remark emphasis the uncertain characteristic of the method.

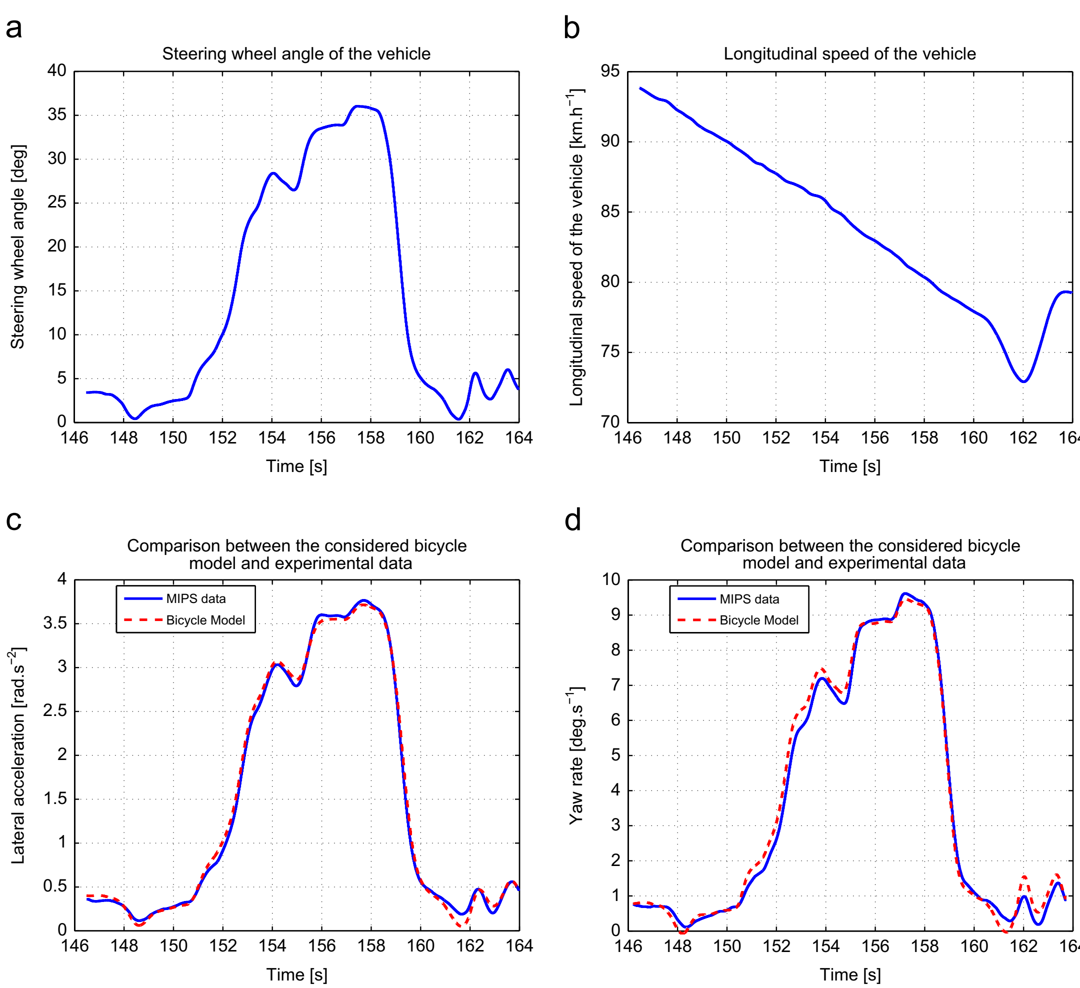

Fig. 4. Comparison between MIPS data and the bicycle model: (a) steering wheel angle, (b) vehicle longitudinal speed, (c) lateral acceleration, (d) yaw rate.

5.3. Fault detection by $\mathcal{U}$−LPV parity space approach

The matrices A_1, B_d and D_d have been designed as in (5) to complete the model which is not perfect. In effect, mostly the stiffness parameters $c_{\alpha V}$ and $c_{\alpha H}$ are not perfectly known. Consequently, the matrix A_1 is chosen as 20% of the nominal state matrix A_0, by considering a constant mean speed of $75\ \text{km h}^{-1} \simeq 20.83\ \text{m s}^{-1}$, so $\rho^* = \frac{1}{20.83} = 0.048$.

$$A_1 = 0.2A_0(\rho^*) = 0.2A_0(0.048)$$

In addition, one uncertain input is considered with its distribution matrices B_d and D_d given by

$$B_d = 0.2B_0(0.048), \quad D_d = \begin{bmatrix} 0 \\ 2.064 \end{bmatrix}$$

Those matrices allow us to encompass the differences observed in Fig. 4. At this step, the system formulation has exactly the same structure as in (5).

The horizon s has been chosen as $s=3$ in order to ensure a perfect decoupling face to the nominal system.

As the scheduling parameter ρ_1 is related to the longitudinal speed of the vehicle, it can be assumed as constant within the horizon of $s=3$ samples (60 ms).

The matrix $P(\rho_k)$ – which constitutes the first step in the computation of the parity matrix – is built from Eq. (12). Then matrices $\Gamma_1(\rho_k)$ and $\Gamma_2(\rho_k)$ are constructed. It yields six products of the scheduling parameters ρ_{1k} in their definition.

The constant κ has been chosen as $\kappa=27$ to guarantee the positive definiteness of the matrix $\Gamma_\kappa(\rho_k)$. Thus, the inverse matrix $\Gamma_k^{-1}(\rho_k)$ is computed. It results 14 products of the scheduling parameter ρ_{1k}.

At this step, for the sake of the polytopic modeling, it should be considered $2^{14} = 16\,384$ vertices of the polytope, which is technically not available. It has to be pointed out that in this applicative example, the scheduling parameter ρ_{1k} is always lower than 1 since the vehicle longitudinal speed is larger than $3.6\ \text{km h}^{-1}$. As a first simplification, the terms ρ_{1k}^x lower than a certain threshold can be omitted. Moreover, thanks to the simplifications exposed in Section 4.6.2, the necessary number of vertices is reduced to 6, which is far easier to compute and to implement.

Finally, the LMI optimization process leads to the vector $W_2(\rho_k)$ constructed with six sub-vectors W_{2i} as

$$W_2(\rho_k) = \sum_{i=1}^{6} \tilde{\alpha}_i(\rho_k) W_{2i} \qquad (49)$$

It is recalled that all those computations are handled off-line.

5.4. Faulty scenario

The data are performed on a healthy measurement system. As a consequence, the fault has been numerically added in simulation. The considered fault $f(k)$ has been added from $t = 150$ s to $t = 158$ s to the first system output $a_y(k)$. Its amplitude is $+0.5\ \text{m s}^{-2}$.

5.5. Residuals

It has been compared both LPV and $\mathcal{U}$−LPV approaches applied in this application. Both residuals r_{LPV} and r_{LPVu} respectively representing the residual of the LPV and $\mathcal{U}$−LPV approach are illustrated in Fig. 5.

Remark 9. It has to be recalled that a residual does not have any dimension. In this case, the comparison between different residuals has been done with normalized residuals, meaning that the mean value of the residual during the faulty time is one.

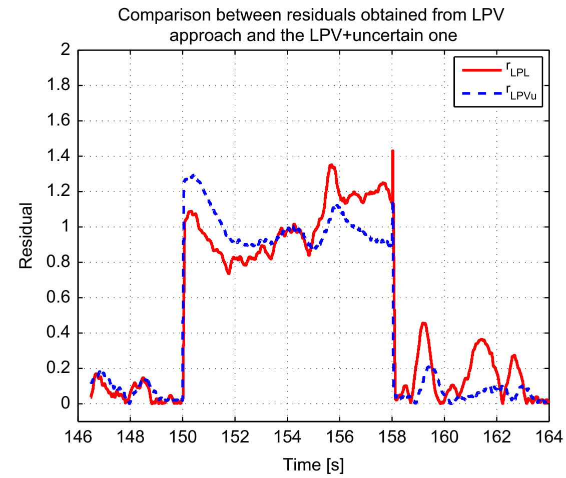

Fig. 5. Comparison between LPV and $\mathcal{U}$−LPV approaches.

In this approach, it can be shown that both approaches lead to effective fault detection. However, the LPV approach is quite sensitive to the unmodeled dynamics. In effect, it can be shown that after the fault, when the residual should be small, it remains some large amplitudes. As an information, the min/max ratio representing the minimum value in faulty case face the maximal value in healthy situation is computed: $\varpi_{LPV} = 1.39$.

On the other hand, the $\mathcal{U}$−LPV approach gives some better results. In effect, it can be shown that the effect of unmodeled dynamics has been attenuated. In effect, the min/max ratio is in this case $\varpi_{LPVu} = 4.04$. This result emphasizes the uncertain characteristic of the approach.

6. Conclusion

The problem which is considered in this paper is the design of residuals for Fault Detection on LPV systems and Uncertain LPV systems subject to Unknown Inputs. The objectives have been fulfilled by considering the classical parity-space based fault detection approach, but addressed for scheduled matrices. The resulting parity matrices are also parameters dependent. Thus, the uncertainty has been tackled by synthesizing a parity matrix via an LMI optimization which is sensitive to the fault and the least receptive to uncertainties nor disturbances.

The method has been successfully applied to experimental data coming from the MIPS laboratory on a vehicle "Renault Scenic". The final result compares the single LPV approach face to the $\mathcal{U}$−LPV one. This second approach shows the interest of considering uncertainties within the residual synthesis.

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