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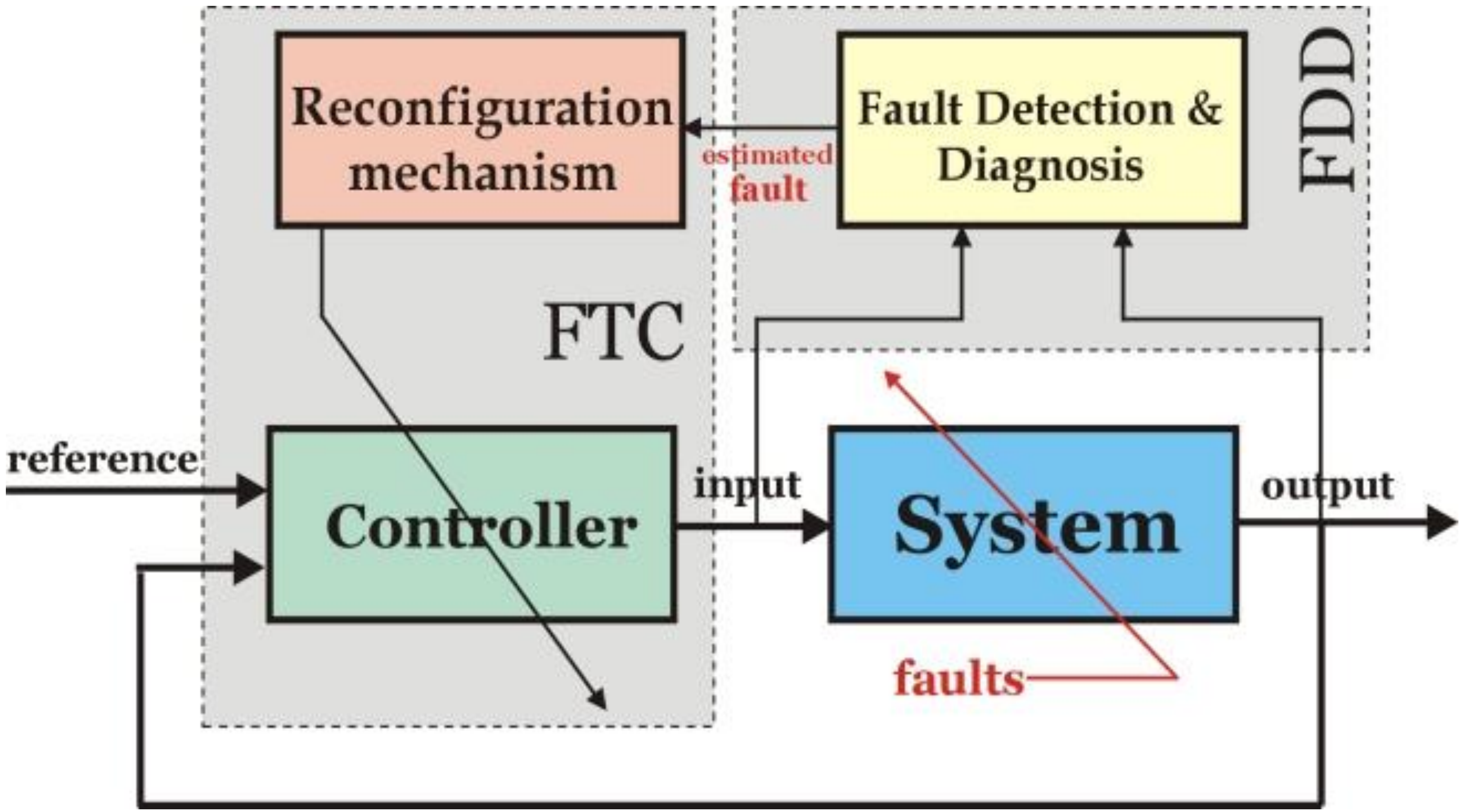
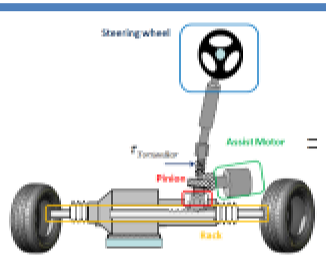
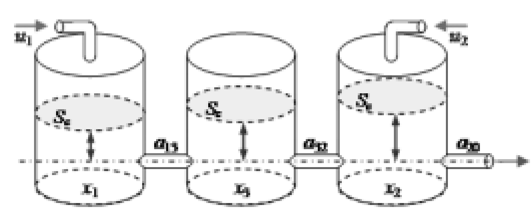
# AC-515 Model based Fault-Diagnosis for Linear Systems



# Damien Koenig

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  - ✓ Grenoble-Inp ESISAR school of Embedded Systems

My main concerns in automatic control are devoted to state estimation, fault diagnosis and fault tolerant control of complex systems.

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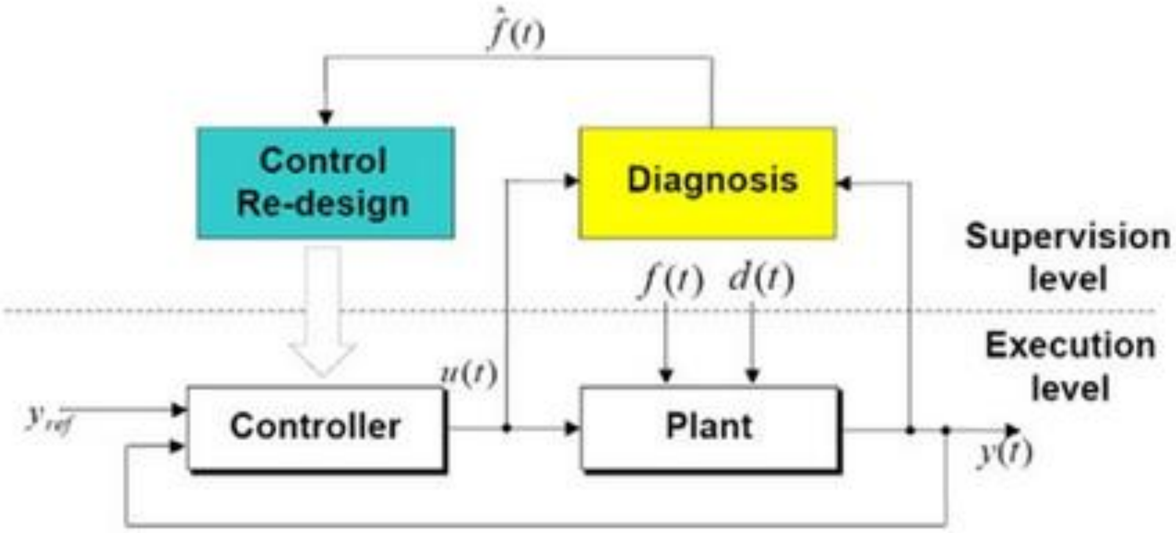
# Nomenclature

- **Functions**
  - ✓ Fault Diagnosis
    - Determination of the kind, size, location and time of detection of a fault. Follows fault detection. Include Fault detection and identification.
  - ✓ Supervision
    - Monitoring a physical and taking appropriate actions to maintain the operation in the case of fault.

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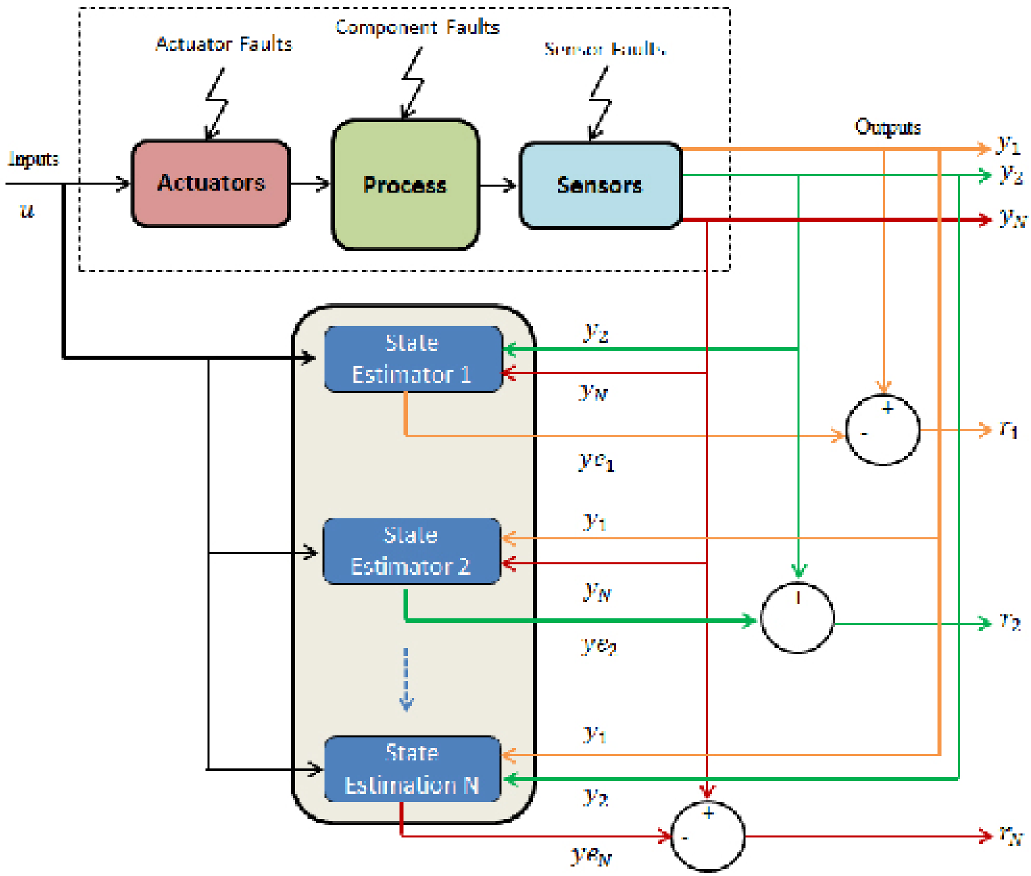
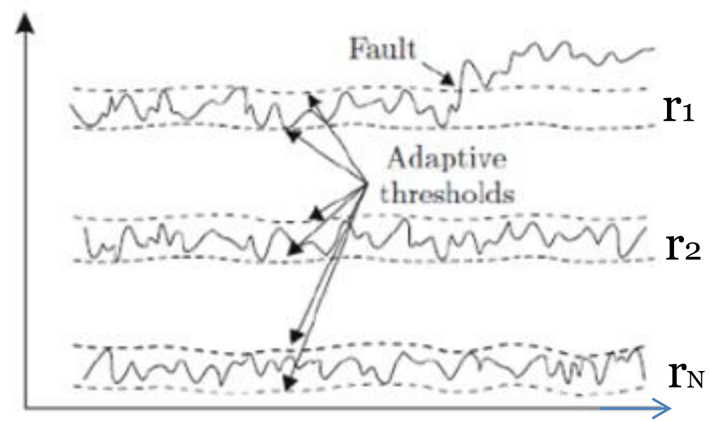
## ❑ Functions

✓ Diagnosis

- Determination of the kind, location and time of detection of a fault. Follows fault detection.

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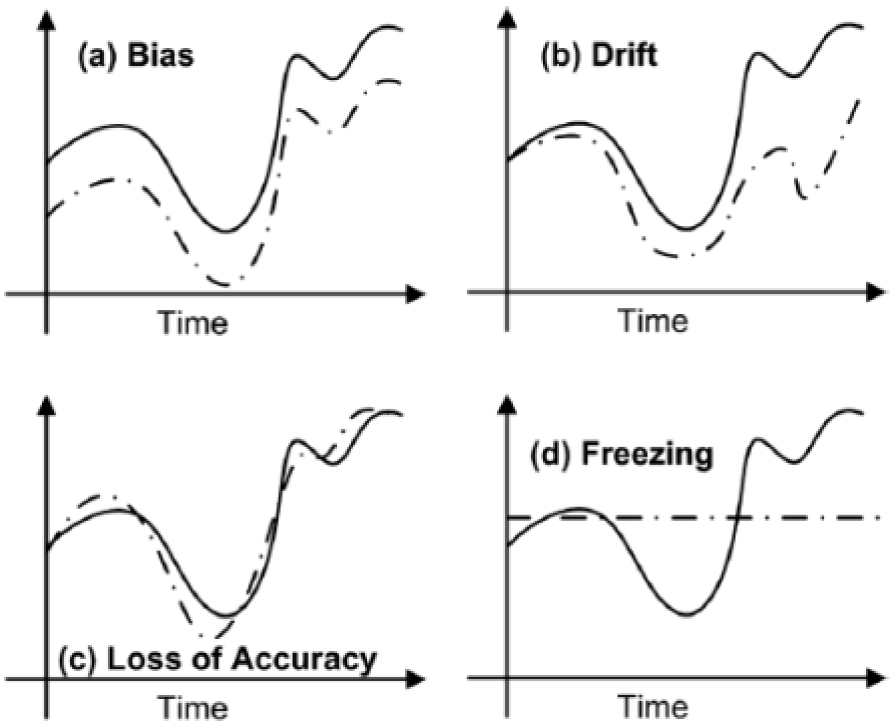
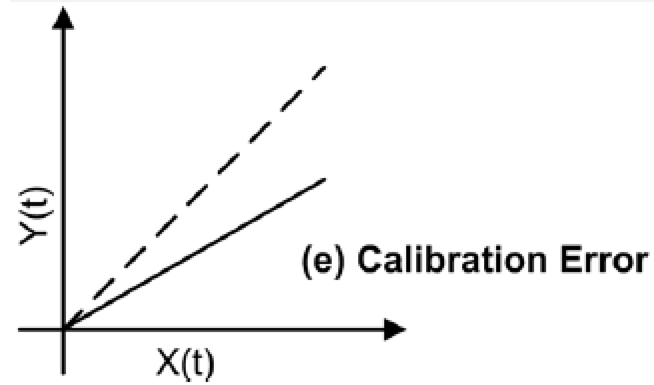
# Example of various sensor faults on system measurements




The figure depicts the effect of the different faults on system measurements.




$$y_i(t) = \begin{cases} x_i(t) & & \forall t \geq t_0 & \text{No failure} \\ x_i(t) + b_i & \dot{b}_i(t) = 0, b_i(t_{Fi}) \neq 0 & & \text{Bias} \\ x_i(t) + b_i(t) & |b_i(t)| = c_i t, 0 < c_i << 1 & \forall t \geq t_{Fi} & \text{Drift} \\ x_i(t) + b_i(t) & |b_i(t)| \leq \bar{b}_i, \dot{b}_t(t) \in L^{\infty} & \forall t \geq t_{Fi} & \text{Loss of accuracy} \\ x_i(t_{Fi}) & & \forall t \geq t_{Fi} & \text{Sensor freezing} \\ k_i(t) x_i & 0 < \bar{k} \leq k_i(t) \leq 1 & \forall t \geq t_{Fi} & \text{Calibration error} \end{cases}$$

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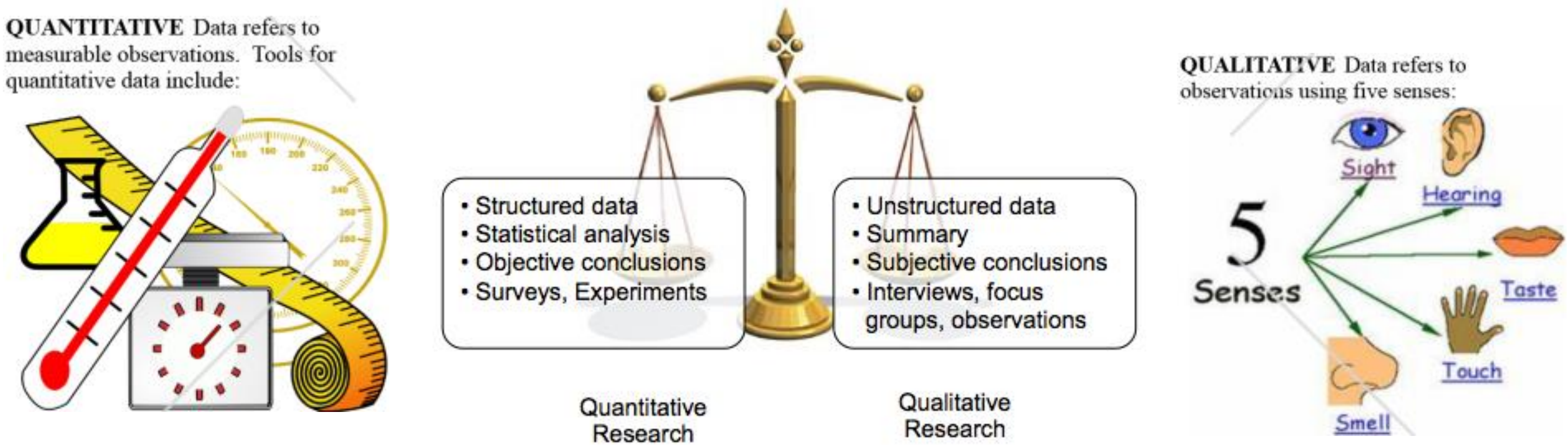
# Nomenclature

## ❑ Models

- ✓ Quantitative model
  - Use of static and dynamic relations among system variables and parameters in order to describe a system's behavior in quantitative mathematical terms.
- ✓ Qualitative model
  - Use of static and dynamic relations among system variables in order to describe a system's behavior in qualitative terms such as causalities and IF-THEN rules.

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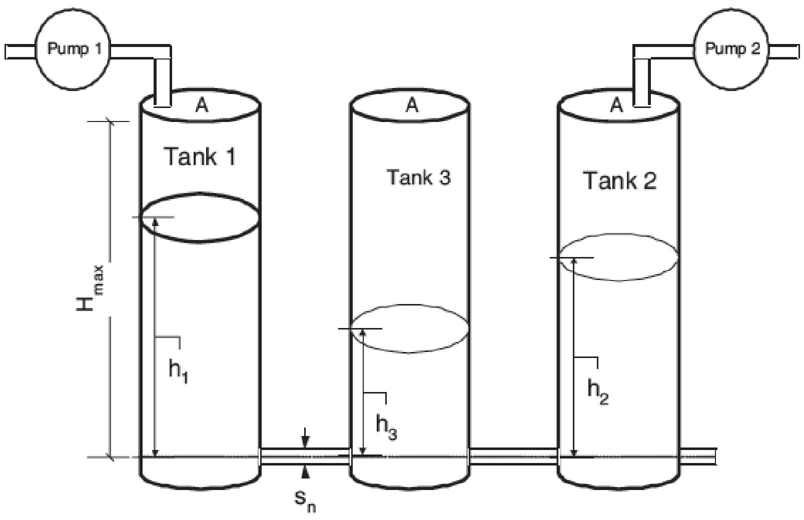
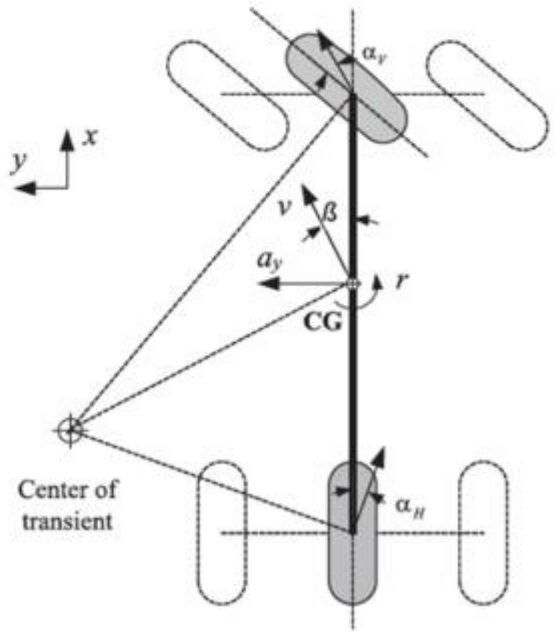
# Application Examples

- Real Processes
- FDI applications

DC motor




Three tank system




Vehicle system

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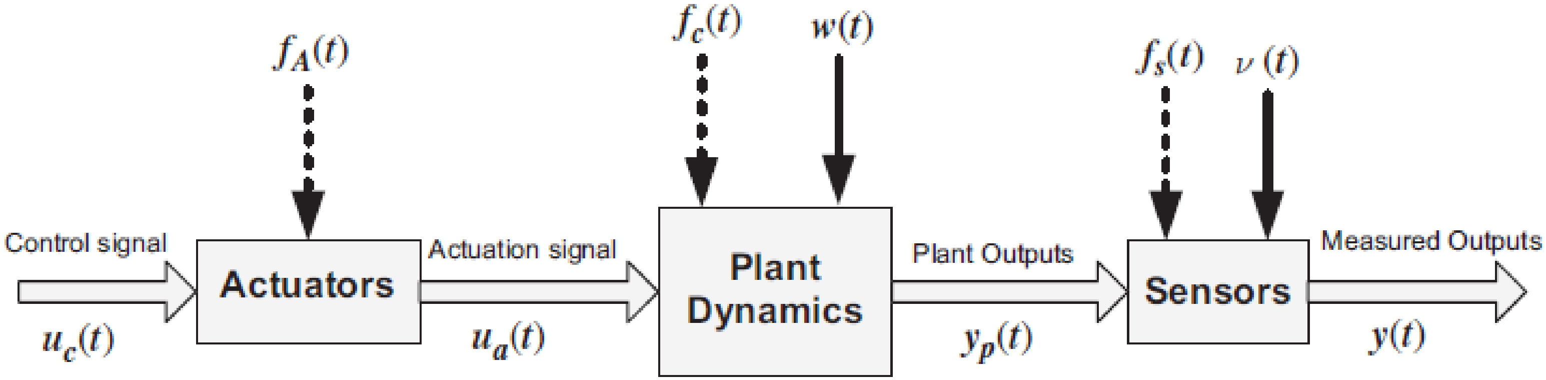
# Usual Fault Diagnosis Decomposition

- The fault system diagnosis can be decomposed into three parts including
  - sensors,
  - actuators,
  - and system dynamics.
- A reliable fault diagnosis system should be able to distinguish faults from system disturbances and measurement noise. More precisely, the fault diagnosis system must be robust to these uncertainties while remaining sensitive to faults.

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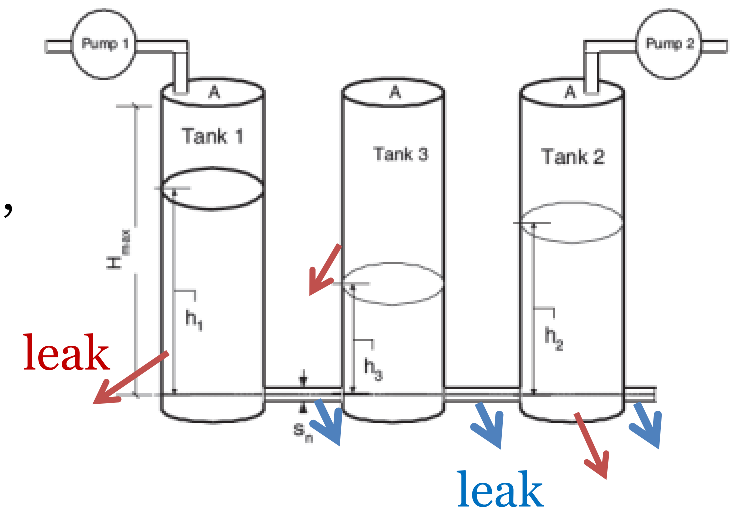
# Fault diagnosis of three tank system

- Modelling of faults




- Three types of faults are considered :
  - ✓ component faults : leaks in the three tanks, which can be modelled as additional mass flows out of tanks,

$$\theta_{A_1}\sqrt{2gh_1}, \theta_{A_2}\sqrt{2gh_2}, \theta_{A_3}\sqrt{2gh_3}$$

where $\theta_{A1}$, $\theta_{A2}$ and $\theta_{A3}$ are unknown and depend on the size of the leaks

- component faults: plugging between two tanks

$$\theta_{A_4} a_1 s_{13} \mathrm{sgn}(h_1 - h_3)\sqrt{2g|h_1 - h_3|}, \theta_{A_6} a_3 s_{23} \mathrm{sgn}(h_3 - h_2)\sqrt{2g|h_3 - h_2|}, \quad \theta_{A_5} a_2 s_0 \sqrt{2gh_2}$$

- sensor faults: three additive faults in the three sensors, denoted by f1, f2 and f3
- actuator faults: faults in pumps, denoted by f4 and f5

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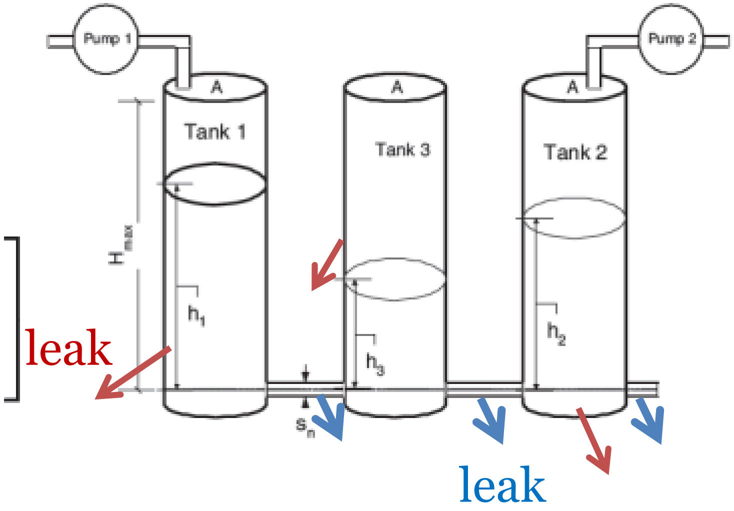
# Fault Model diagnosis of three tank

- They are modelled as follows




$$\dot{x} = (A + \Delta A_F)\, x + Bu + E_f f, y = Cx + F_f f$$

$$\Delta A_F = \sum_{i=1}^{6} A_i \theta_{A_i}, A_1 \begin{bmatrix} -0.0214 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.0371 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.0262 \end{bmatrix}, A_4 = \begin{bmatrix} -0.0085 & 0 & 0.0085 \\ 0 & 0 & 0 \\ 0.0085 & 0 & -0.0085 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.0111 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.0084 & 0.0084 \\ 0 & 0.0084 & -0.0084 \end{bmatrix}, f = \begin{bmatrix} f_1 \\ \vdots \\ f_5 \end{bmatrix}$$

$$E_f = \begin{bmatrix} 0 & B \end{bmatrix} \in \mathcal{R}^{3\times 5}, F_f = \begin{bmatrix} I_{3\times 3} & 0 \end{bmatrix} \in \mathcal{R}^{3\times 5}.$$

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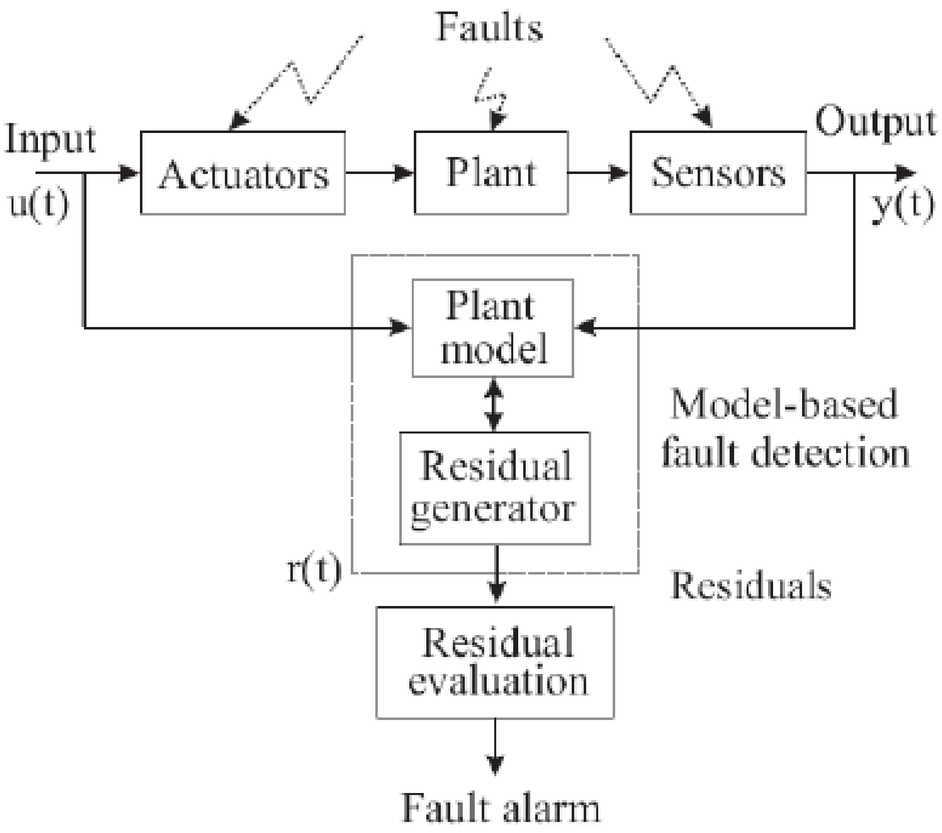
# Method of Residual Generation

- The most relevant analytical model-based residual generation methods developed have been divided into three categories:
  - ✓ Parity space approach
  - ✓ Observer-based approach
  - ✓ Parameter estimation approach

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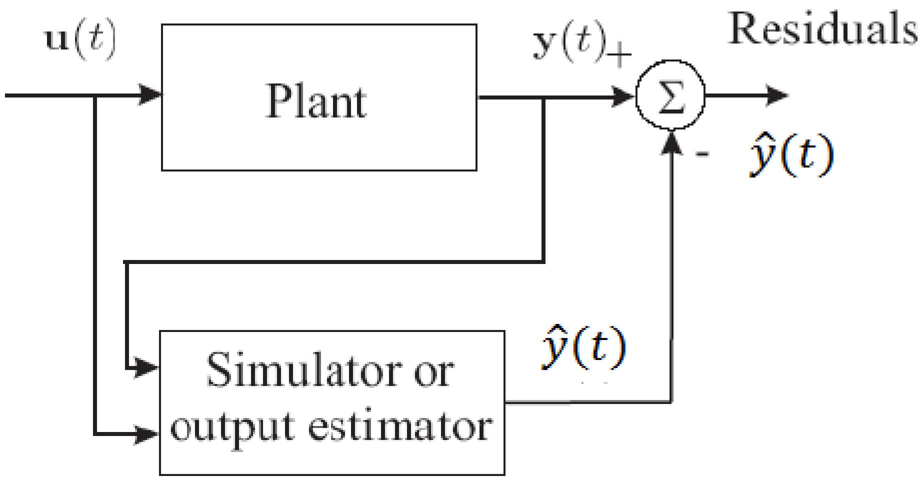
# Residual Generator Structure

- Residual generation via system simulator




$r(t) = y(t) - \hat{y}(t)$

$\hat{y}(t)$ is the estimated plant output

- Simulator is usually an Observer, KF or UIO

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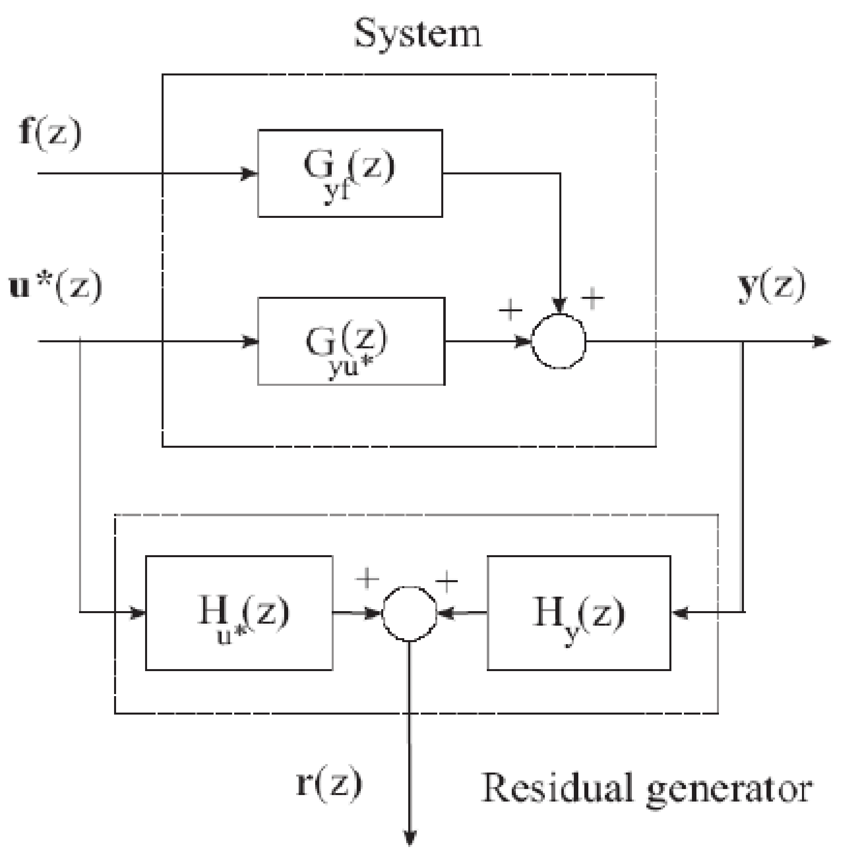
# Residual Generator Structure

- Residual generator:

$$y(z) = G_{yf}(z)f(z)+G_{yu^*}(z)u^*(z)$$

$$r(z) = H_{u*}(z)u^*(z)+H_y(z)y\,(z)$$

r(t)=0 if and only if f(t) = 0




- Constraint conditions: *design*

$$H_{u*}(z) + H_y(z)G_{yu^*}(z)=0$$

$$H_y(z)G_{yf}(z)\neq 0$$

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# Change Detection (Example 2)

- Residual Evaluation

$$\begin{cases} J(\mathbf{r}(t)) \leq \varepsilon(t) & \text{for} \quad \mathbf{f}(t) = 0 \\ J(\mathbf{r}(t)) > \varepsilon(t) & \text{for} \quad \mathbf{f}(t) \neq 0 \end{cases}$$

$$J(r(t)) \equiv |r(t)|$$

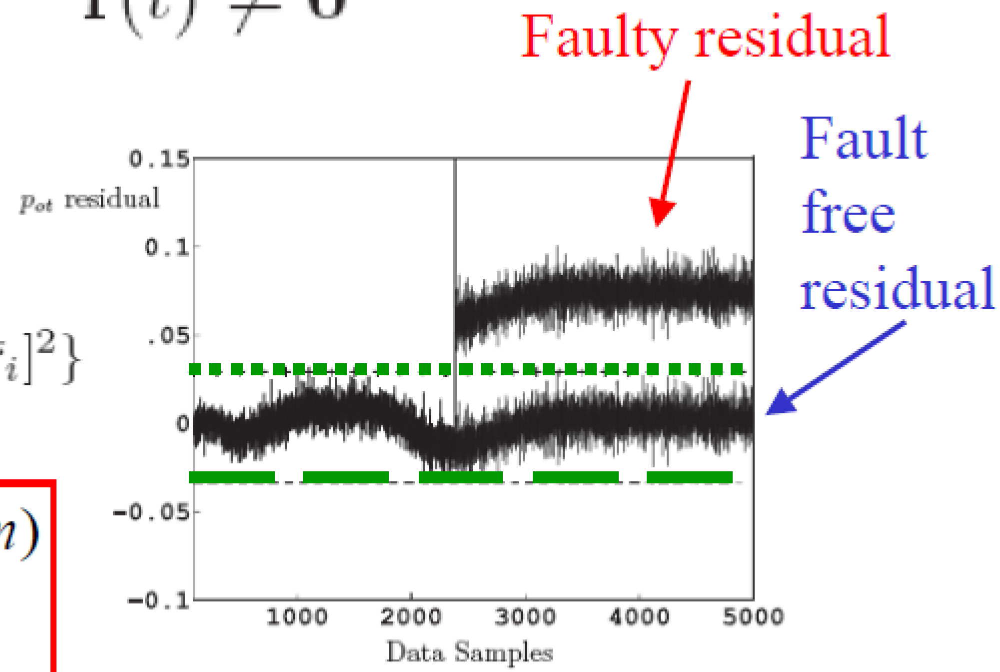
Detection thresholds

ε(t)

$$\bar{r}_i = E\{r_i(t)\}; \qquad \bar{\sigma}_i^2 = E\{[r_i(t) - \bar{r}_i]^2\}$$

$$\varepsilon(t) = \bar{r}_i \pm \delta \times \bar{\sigma}_i \qquad (i = 1, \cdots, m)$$

with $\delta \geq 3$

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# Residual Generation Techniques

- ❑ Observer-based approaches
- ❑ Parity (vector) relations
- ❑ Fault detection via parameter estimation

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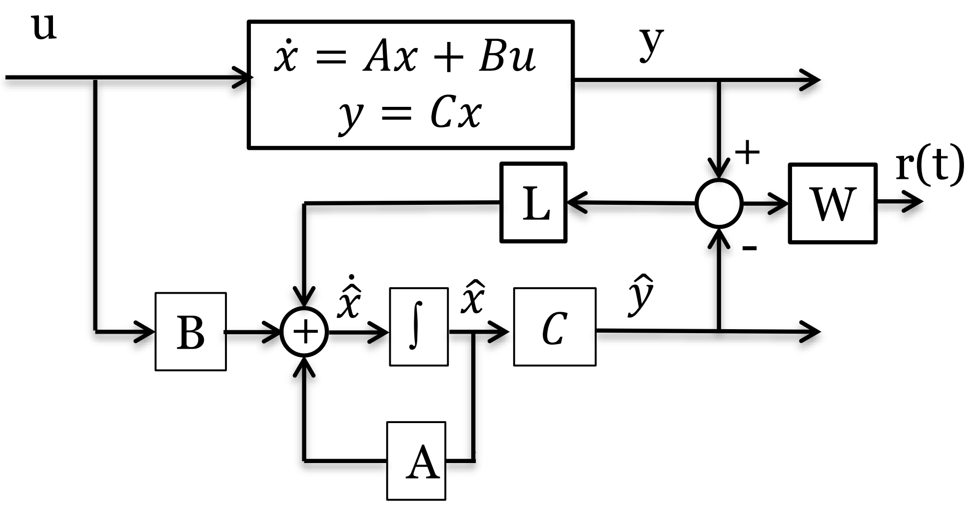
# Observer-based approaches




## ❑ Residual General Structure

- ✓ Plant model without fault
  - $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$
- ✓ Observer model
  - $\{\dot{\hat{x}} = A\hat{x}$ + Bu + L(y-C$\hat{x}$)
- ✓ Dynamic of the state estimation $e_x = x - \hat{x}$
  - $\dot{e_x}$=(A-LC) $e_x$
- ✓ State estimation error property
  - $\lim_{t \to \infty} e_x = 0 \quad for\ fault\ free$
- ✓ Residual
  - r(t) = W(y(t)-$\hat{y}(t)$) $= WCe_x(t)$

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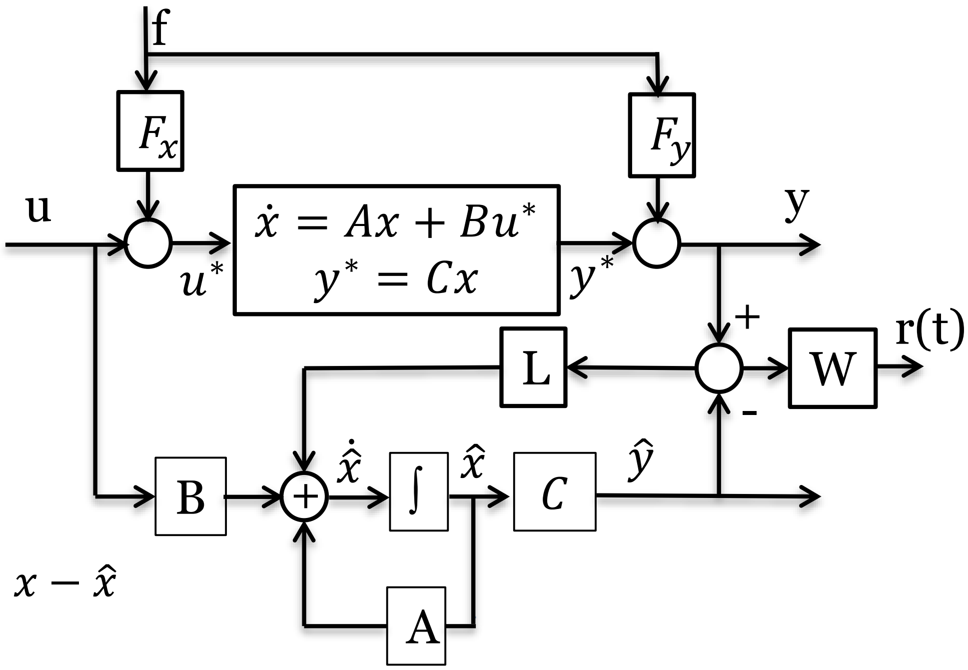
# Observer-based approaches




## ❑ Residual General Structure

- ✓ Plant model with fault
  - $\begin{cases} \dot{x} = Ax + Bu + F_x \text{f} \\ y = Cx + F_s \text{f} \end{cases}$
- ✓ Observer model
  - $\begin{cases} \dot{\hat{x}} = A\hat{x} + \text{Bu} + \text{L}(\text{y} - \text{C}\hat{x}) \\ \hat{y} = C\hat{x} \end{cases}$
- ✓ Dynamic of the state estimation $e_x = x - \hat{x}$
  - $\dot{e_x}$=(A-LC) $e_x$+ $F_x$f −L$F_s$f
- ✓ Output state estimation error
  - $\tilde{y}$(t)=y(t)-$\hat{y}(t)$=C$e_x(t) + F_s$f(t)
- ✓ State estimation error property
  - $\lim_{t\to\infty} e_x \neq 0$ *when a fault occur*
- ✓ Residual
  - r(t) = W(y(t)-$\hat{y}(t)$) = $WCe_x(t)$+W$F_s$f(t)
  - $\lim_{t\to\infty} r(t) = \left(-WC\,(A - LC)^{-1}(F_x - LF_s) + \text{W}F_s\right) f(t) \neq 0$ *when a fault occur*

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# Basic principles of fault detection filters (BFDF)

- In order to describe the BFDF, the simple fault model is used
  - $\begin{cases} \dot{x} = Ax + Bu + b_i f_{ai}(t) \\ y = Cx + I_j f_{sj}(t) \end{cases}$
- ✓ Where
  - $b_i f_{ai}(t)$ (i=1,2, …, r) denotes that a fault occurred in the $i_{th}$ $actuator, b_i \in R^n$ is the $i_{th}$ column of the input matrix B and $f_{ai}(t)$ is an unknown scalar time-varying function which represents the evolution of the fault.
  - $I_j f_{sj}(t)$ (j=1,2, …, m) denotes that a fault occurs in the $j_{th}$ $sensor, I_j \in R^m$ is a unit vector corresponding to a fault occurs in the $j_{th}$ $sensor$.
- ✓ BFDF
  - $\{\dot{\hat{x}} = A\hat{x}$ + Bu + L(y-C$\hat{x}$)
- ✓ State estimation error property
  - $\dot{e_x}$=(A-LC) $e_x + b_i f_{ai}(t) - \mathrm{L} I_j f_{sj}(t)$
- ✓ Output state estimation error
  - $\tilde{y}$(t)=C$e_x(t) + I_j f_{sj}(t)$
- ✓ Residual
  - $\lim_{t \to \infty} r(t) \neq 0$ $when\ fault\ sensor\ or\ actuator\ occur$

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# Case 1 : Simultaneous faults sensor

- The sensor fault model is described by
  - $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + f_s(t) \end{cases}$
  - ✓ Where
    - $f_s(t)$ denotes the faults sensor vector
- We build a bank of Observers, where each observer i is piloted by sensor $y_i$
  - ✓ $\{\dot{\hat{x}} = A\hat{x} + \mathrm{Bu} + L_i(y_i - C_i\hat{x})$
  - ✓ State estimation error property
    - $\dot{e_x} = (\mathrm{A} - L_i C_i)\, e_x - L_i f_{si}(t)$
  - ✓ Output state estimation error
    - $r_i(\mathrm{t}) = y_i - \hat{y}_i = C_i \; e_x(t) + f_{si}(t)$
  - ✓ Residual
    - $\lim_{t \to \infty} r_i(t) \neq 0$ *when* $f_{si}(t)$ hold true
    - $r_i(t) \in \Re$

$i^{th}$ sensor fault model

$$\begin{cases} \dot{x} = Ax + Bu \\ y_i = C_i x + f_{si}(t) \end{cases}$$

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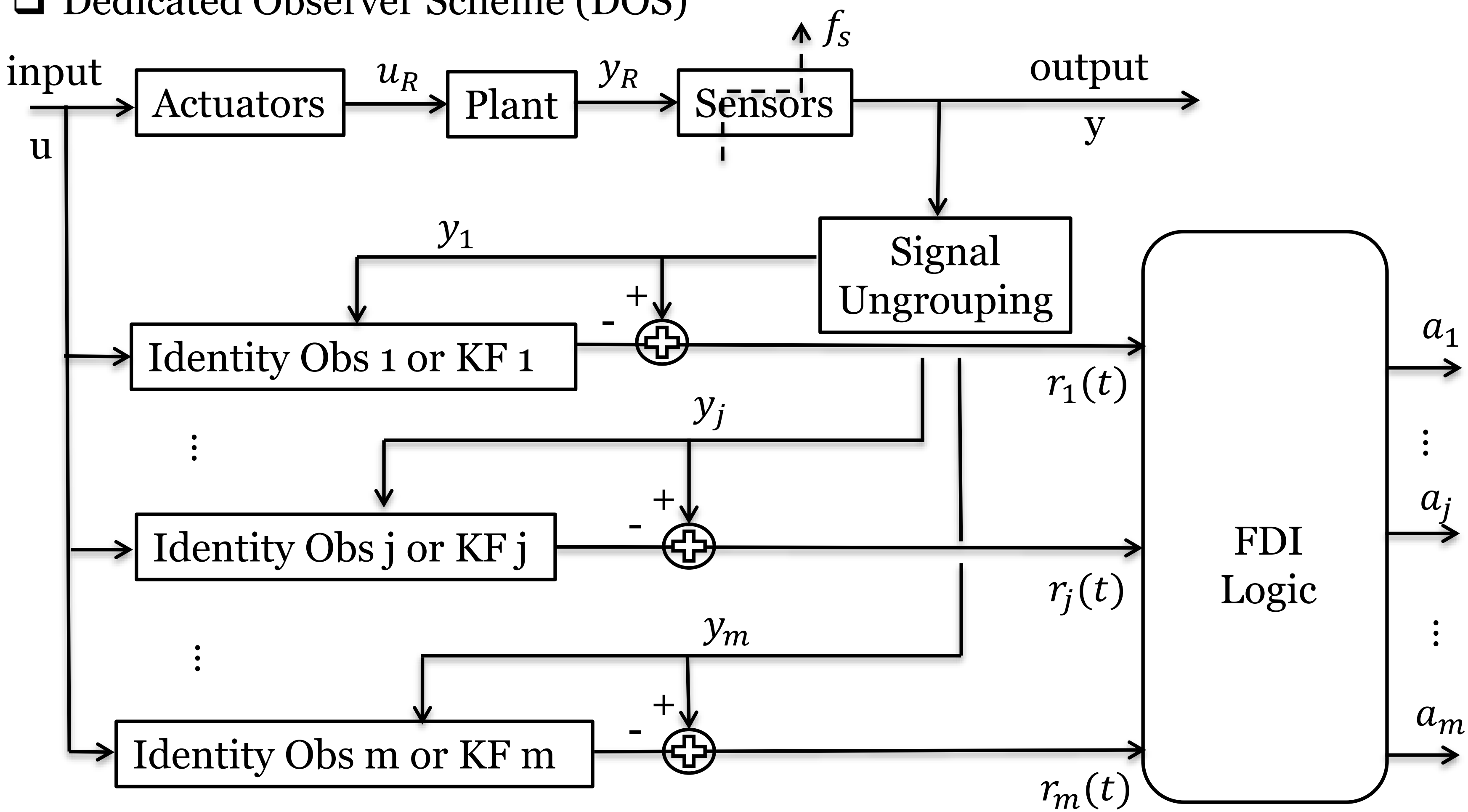
# Case 1 : Multiple sensors faults isolation scheme

- Dedicated Observer Scheme (DOS)




The number of these observers (estimators) is equal to the number $m$ of system outputs, and each device is driven by a single output and all the inputs of the system. In this case a fault on the $i_{th}$ output affects only the residual function of the output observer or filter driven by the $i_{th}$ output.

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# Case 1 : A simple threshold logic for DOS

- A simple threshold logic can be used to make decision about the appearance of a specific fault by the logic decision according to:
  - ✓ {$\| r_i$ (t)$\| \geq \varepsilon_i$ *implies* $f_{si} \neq 0$
- This isolable residual structure is very simple and all faults can be detected simultaneously, however it is difficult to design in practice.

| | $r_1$ | $r_2$ | … | $r_m$ | |
|---|---|---|---|---|---|
| $f_{s1}$ | 1 | 0 | … | 0 | ⇨ $a_1$=1 if $f_{s1}$ hold true |
| $f_{s2}$ | 0 | 1 | | 0 | |
| ⋮ | | | ⋱ | 0 | |
| $f_{sm}$ | 0 | 0 | | 1 | ⇨ $a_m$=1 if $f_{sm}$ hold true |

Table 2 : DOS

In the tables above, a "1" in $i_{th}$ row and $i_{th}$ column denotes that the residual $r_i$ is sensitive to the fault $f_{si}$, whilst a "0" denotes insensitivity.

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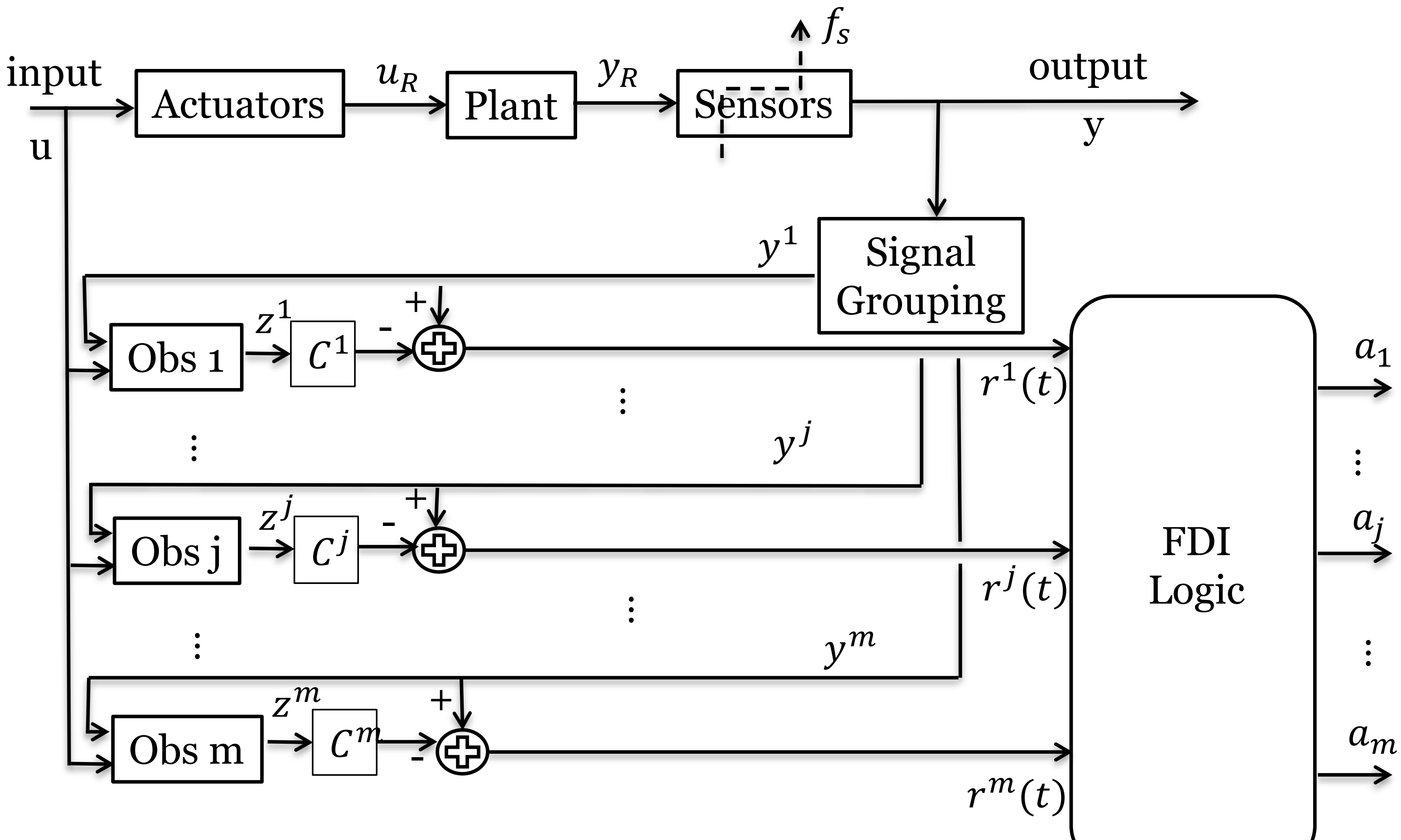
# Case 2 : No simultaneous sensors faults




Where $r^i(t) \in \Re^{m-1}$ i=1, 2 ...m

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# *Case 2 : Sensor Generalized Observer Scheme*

- Analysis of the fault sensor isolability

| | $r^1$ | $r^2$ | … | $r^j$ | … | $r^m$ |
|---|---|---|---|---|---|---|
| $f_{s1}$ | 0 | 1 | 1 | | … | 1 |
| $f_{s2}$ | 1 | 0 | 1 | | … | 1 |
| ⋮ | | | ⋱ | | | ⋮ |
| $f_{sj}$ | 1 | 1 | | 0 | | 1 |
| ⋮ | | | | | ⋱ | |
| $f_{sm}$ | 1 | 1 | 1 | | | 0 |

⇨ $a_1$=1 if $f_{s1}$ hold true

Table 1: *Generalized Observer Scheme* GOS

In the table above, a "1" in $i_{th}$ row and $i_{th}$ column denotes that the residual $r^i$ is sensitive to the fault $f_{si}$ ,whilst a "0" denotes insensitivity.

- Fault alarm
  - $a_1 = L(\|\bar{r}^1\|) \cap L(\|r^2\|) \cap L(\|r^3\|) \cap \ldots \cap L(\|r^m\|)$

where $L(\|r^i\|)=1$ if $\|r^i\| \geq \varepsilon^i$ (L=binary logic function)

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# Robust FDI schemes based on UIOs

- The main task of robust FDI is to generate a residual signal which is robust to the system uncertainty.
- According to the previous discussion, a system with possible sensor and actuator faults can be described as :
  - $\begin{cases} \dot{x} = Ax + Bu + Ed(t) + Bf_a(t) \\ y = Cx + f_s\,(t) \end{cases}$
- ✓ Where
  - $f_a(t) \in \Re^r$ is the actuator faults
  - $f_s(t) \in \Re^m$ is the sensor faults
  - $d \in \Re^q$ is the unknown input (or disturbance)
- To generate a robust residual an UIO is designed :
  - $\begin{cases} \dot{z}(t) = Fz(t) + TBu(\text{t}) + Gy(t) \\ \hat{x}(t) = z(t) + Ny(t) \end{cases}$
- ✓ Where $\hat{x} \in \Re^n$ is the estimated state vector and z$\in \Re^n$ is the state of the full order observer, and F, T G, N are matrices to be designed for achieving unknown input de-coupling and other design requirements.
- ✓ Residual
  - r(t)=y(t)-C$\hat{x}(t)$

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# UIO observer design

- When the UIO-based residual is applied to the previous system, the residual and state estimation errors become:
  - ✓ $e_x(t) = x(t) - \hat{x}(t)$=(I-NC)x- $z(t) - Nf_s\,(t)$ = Tx(t) − z(t) − N $f_s\,(t)$
  - ✓ r(t)=y(t)-C$\hat{x}(t) = Ce_x + f_s\,(t)$
  - ✓ Where T=I-NC
- Dynamic of the state estimation error
  - ✓ $\dot{e}_x = \mathrm{T}\dot{x}(\mathrm{t}) - \dot{z}(\mathrm{t}) - \mathrm{N}\dot{f}_s(t) = (TA - GC)x + \mathrm{T}Ed(t) + TBf_a(t) - Fz(t) - Gf_s\,(t) - \mathrm{N}\dot{f}_s(t)$
  - ✓ $\dot{e}_x = (TA - GC)x + \mathrm{T}Ed(t) + TBf_a(t) - F(\mathrm{Tx(t)} - \mathrm{N}\,f_s\,(t) - e_x) - Gf_s\,(t) - \mathrm{N}\dot{f}_s(t)$
  - ✓ $\dot{e}_x = \mathrm{F}\,e_x + (TA - GC - FT)x + \mathrm{T}Ed(t) + TBf_a(t) + (F\mathrm{N} - \mathrm{G})f_s\,(t) - \mathrm{N}\dot{f}_s(t)$
- If one can make the following relations hold true
  - ✓ $TA - GC - FT$=0
  - ✓ T=I-NC
  - ✓ K= -($F$N−G)=G−FN
  - ✓ TE=0
- The state estimation error will then be:
  - ✓ $\dot{e}_x = \mathrm{F}\,e_x + TBf_a(t) - Kf_s\,(t) - \mathrm{N}\dot{f}_s(t)$
  - ✓ If all eigenvalues of F are stable, then
    - $\left\|\lim_{t\to\infty} r(t)\right\| <$ Threshold for fault free case
    - $\left\|\lim_{t\to\infty} r(t)\right\| \geq$ Threshold for faulty case

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# UIO design procedure

- ✓ 1) Check the UI decoupled condition
  - Rank(CE)=Rank(E)=dim(d)
- ✓ 2) Compute
  - TE=0 ⇔ (I-NC)E=0 ⇔ E=NCE ⇔N=E(CE)$^+$=E$\left[(CE)(CE)^T\right]^{-1}(CE)^T$
  - Deduce N and T=I-NC
- ✓ 3) Solve the Syvester equation
  - $TA - GC - FT$=0 ⇔TA-GC-F(I-NC)=0⇔F=TA-(G-FN)C ⇔F=TA-KC
  - Check the observabilty (or detectability) of the pair(TA, C), if (TA, C) observable (detectable), a UIO exists and K can be computed using pole placement or LQ method.
  - Deduce K, F, G=K+FN

❑ The UIO-based residual is also reduced to:

- $\dot{e}_x$= F $e_x + TBf_a(t) - Kf_s\,(t) -$ N$\dot{f}_s(t)$
- r(t)=$Ce_x + f_s\,(t)$

- ✓ It can be seen that the disturbance effects have been de-coupled from the residual.
- ✓ To detect actuator faults, one has to make:
  - TB≠0
- ✓ More specifically, the fault in the $i_{th}$ actuator will affect the residual iff:
  - T$b_i$≠0
- ✓ Where $b_i$ is the $i_{th}$ column of the matrix B.

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# Robust fault isolation schemes based on UIOs

- The fault isolation problem is to located the fault, i.e., to determine in which sensor (or actuator) fault has occurred.

- The sensitivity and insensitivity properties makes isolation possible.

- The ideal situation is to make each residual only sensitive to a particular fault and insensitive to all other faults. However, this ideal situation is normally difficult to achieve.

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# Robust sensor fault isolation schemes

- To design robust sensor fault isolation schemes , all actuators are assumed to be fault-free and the system can be described as:
  - $$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \\ y^j(t) = C^j x(t) + f^j{}_s(t) \\ y_j(t) = c_j x(t) + f_{sj}(t) \end{cases}$$ for j=1, 2, …, m
  - ✓ Where
    - $c_j \in \Re^{1\times n}$ is the $j_{th}\ row\ of\ the\ matrix\ C$
    - $C^j \in \Re^{(m-1)\times n}$ is obtained from the matrix C by deleting $j_{th}\ row c_j$
    - $y_j(t)$ is the $j_{th}$ component of y
    - $y^j(t) \in \Re^{m-1}$ is obtained from the vector y by deleting $j_{th}\ component\ y_j$
- Based on the this description, m UIO-based residual generator can be constructed as:
  - $$\begin{cases} \dot{z}^j(t) = F^j z^j(t) + T^j Bu(t) + G^j y^j(t) \\ \hat{x}(t) = z^j(t) + N^j y^j(t) \\ r^j(t) = y^j(t) - C^j\hat{x}(t) = (I - C^j N^j) y^j(t) - C^j z^j(t) \end{cases}$$ for j=1, 2, …, m
  - ✓ Where $r^j(t) \in \Re^{m-1}$

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# Robust sensor fault isolation schemes

- In order to obtain an UIO, the parameter matrices must satisfy the following equalities:
  - ✓ $N^j = \mathrm{E}(C^j\mathrm{E})^{+} = \mathrm{E}\left[(C^jE)(C^jE)^T\right]^{-1}(C^jE)^T$
  - ✓ $T^j = \mathrm{I} - N^j C^j$
  - ✓ $F^j = T^j\mathrm{A} - K^j C^j$ to be stabilized
  - ✓ $G^j = K^j + F^j N^j$
  - ✓ for j=1, 2, …, m

$$\begin{cases} \dot{z}^j(t) = F^j z^j(t) + T^j Bu(t) + G^j y^j(t) \\ \hat{x}(t) = z^j(t) + N^j(t) y^j(t) \\ r^j(t) = y^j(t) - C^j \hat{x}(t) \end{cases}$$

- When all actuators are fault free and a fault occurs in the $j_{th}$ sensor, the residual will satisfy the following isolation logic:
  - ✓ $\begin{cases} \|r^j(t)\| < \varepsilon^j \\ \|r^k(t)\| \geq \varepsilon^k \end{cases}$ for k=1, …, j-1, j+1, …,m
  - ✓ Where $\varepsilon^j$ (j=1, 2…,m) are isolation threshold.

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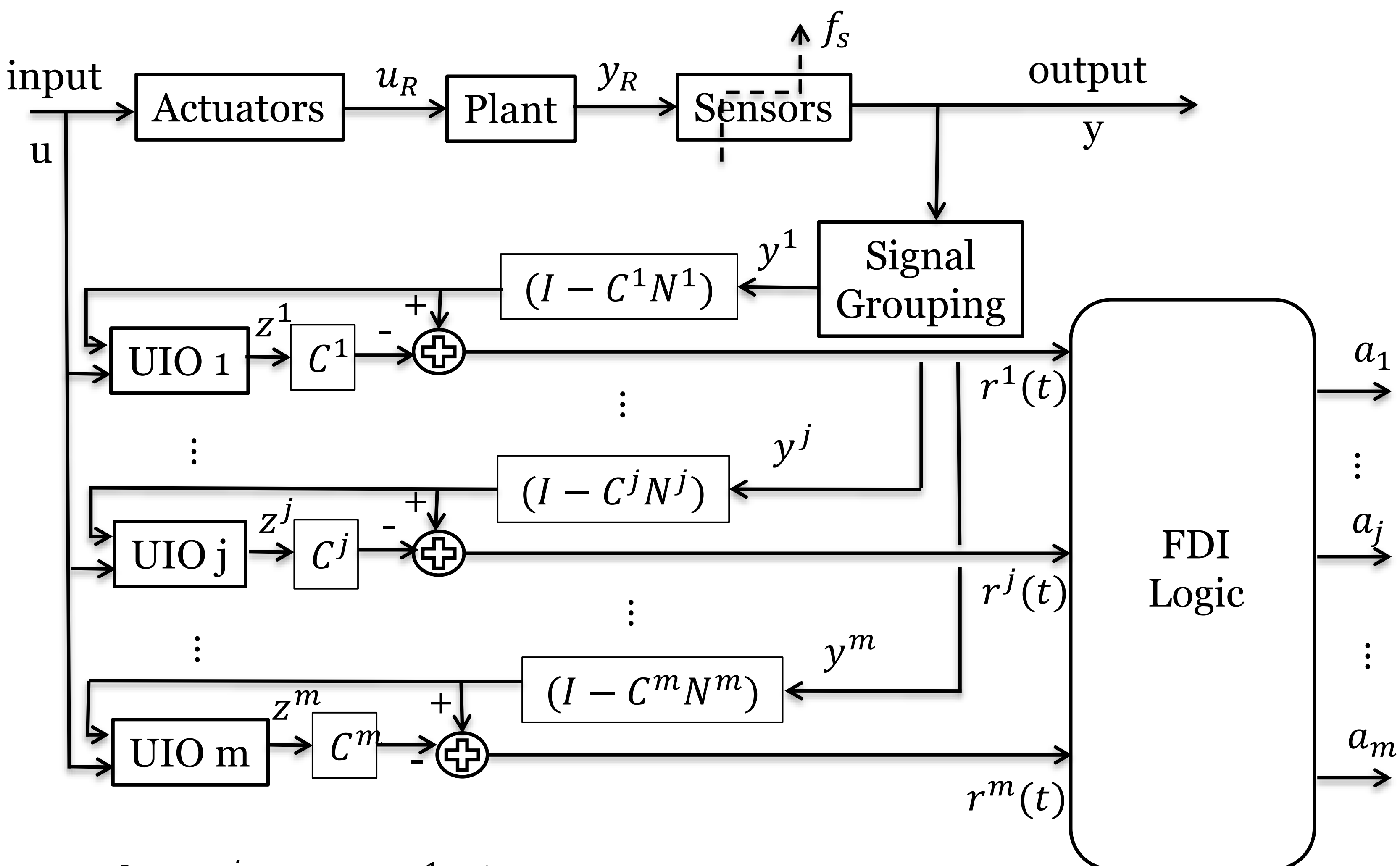
NANOELEC

# A robust sensor fault isolation scheme




Where $r^i(t) \in \Re^{m-1}$ i=1, 2 …m

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# Sensor Generalized Observer Scheme

❑ Analysis of the fault sensor isolability

| | $r^1$ | $r^2$ | … | $r^j$ | … | $r^m$ |
|---|---|---|---|---|---|---|
| $f_{s1}$ | 0 | 1 | 1 | | … | 1 |
| $f_{s2}$ | 1 | 0 | 1 | | … | 1 |
| ⋮ | | | ⋱ | | | ⋮ |
| $f_{sj}$ | 1 | 1 | | 0 | | 1 |
| ⋮ | | | | | ⋱ | |
| $f_{sm}$ | 1 | 1 | 1 | | | 0 |

⇨ $a_1$=1 if $f_{s1}$ hold true

Table 1: *Generalized Observer Scheme* GOS

In the table above, a "1" in $i_{th}$ row and $i_{th}$ column denotes that the residual $r^i$ is sensitive to the fault $f_{si}$ ,whilst a "0" denotes insensitivity.

❑ Fault alarm

- $a_1 = L(\|\bar{r}^1\|) \cap L(\|r^2\|) \cap L(\|r^3\|) \cap \ldots \cap L(\|r^m\|)$

where $L(\|r^i\|)=1$ if $\|r^i\| \geq \varepsilon^i$ (L=binary logic function)

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# Robust sensor fault isolation schemes

- Existence condition : In order to satisfy the constraints
  - ✓ $T^j$E=0 $c_1$ : decoupled condition
  - ✓ $F^j$=$T^j$A- $K^j$ $C^j$ stable $c_2$ : detectability condition

the following rank conditions must be verified for each observer :

- ✓ $c_1$: $\text{rank}\left(\begin{bmatrix} C^j\,\text{E} \\ E \end{bmatrix}\right)$=rank$(C^j\ \text{E})$=dim(d)
- ✓ $c_2$: rank $\left(\begin{bmatrix} pI_n - T^jA \\ C^j \end{bmatrix}\right)$=n $\forall\ \Re(\lambda(T^jA))\geq 0$
- ✓ Proof of $c_1$ : $T^j$ E=0 ⇔ (I- $N^j$ $C^j$ )E=0 ⇔ $N^j$ $C^j$ E=E
  - The solution of $N^j$ $C^j$ E=E depends on the rank of matrix $C^j$ E.
  - A solution exists iff : $\text{rank}\left(\begin{bmatrix} C^j\,\text{E} \\ E \end{bmatrix}\right)$=rank$(C^j\ \text{E})$=dim(d) [Rao]
  - C. R. Rao and S. K. Mitra, *Generalized Inverse of Matrices and Its Applications*. New York: Wiley, 1971.

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# Robust actuator fault isolation schemes

- To design robust actuator fault isolation schemes, all sensors are assumed to be fault-free and the system can be described as:

  - $$\begin{cases} \dot{x}(t) = Ax(t) + B^i u^i(t) + B^i f^i{}_a(t) + b_i(u_i(t) + f_{ai}(t)) + Ed(t) \\ \qquad = Ax(t) + B^i u^i(t) + B^i f^i{}_a(t) + E^i d^i(t) \\ y(t) = Cx(t) \\ for \;\; i = 1, 2, \ldots, r \end{cases}$$

  ✓ Where

  - $b_i \in \Re^n$ is the $i_{th}$ $column\ of\ the\ matrix\ B$
  - $B^i \in \Re^{n\times(r-1)}$ is obtained from the matrix B by deleting $i_{th} column\ B_i$
  - $u_i(t)$ is the $i_{th}$ component of u
  - $u^i(t) \in \Re^{r-1}$ is obtained from the vector u by deleting $i_{th}\ component\ u_i$
  - $E^i$=$[E \quad b_i]$
  - $d^i(t)$=$\begin{bmatrix} d(t) \\ u_i(t) + f_{ai}(t) \end{bmatrix}$

- Based on the this description, r UIO-based residual generator can be constructed as:

  - $$\begin{cases} \dot{z}^i(t) = F^i z^i(t) + T^i B^i u^i(t) + G^i y(t) \\ \hat{x}(t) = z^i(t) + N^i y(t) \\ r^i(t) = y(t) - C^i \hat{x}(t) = (I - CN^i) y(t) - Cz^i(t) \end{cases}$$ for i=1, 2, ..., r

  ✓ Where $r^i(t) \in \Re^m$

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# Robust actuator fault isolation schemes

- In order to obtain an UIO, the parameter matrices must satisfy the following equalities:
  - $N^i = E^i(CE^i)^+$
  - $T^i$=I- $N^i C$
  - $F^i$=$T^i$A- $K^i C$ to be stabilized
  - $G^i = K^i + F^i N^i$
  - for i=1, 2, …, r

$$\begin{cases} \dot{z}^i(t) = F^i z^i(t) + T^i B^i u^i(t) + G^i y(t) \\ r^i(t) = \left(I - CN^i\right) y(t) - C z^i(t) \end{cases}$$

- When all sensors are fault free and a fault occurs in the $i_{th}$ actuator, the residual will satisfy the following isolation logic:
  - $\begin{cases} \|r^i(t)\| < \varepsilon^i \\ \|r^k(t)\| \geq \varepsilon^k \end{cases}$ for k=1, …, i-1, i+1, …,m
  - Where $\varepsilon^i$ (i=1, 2…,r) are isolation threshold.

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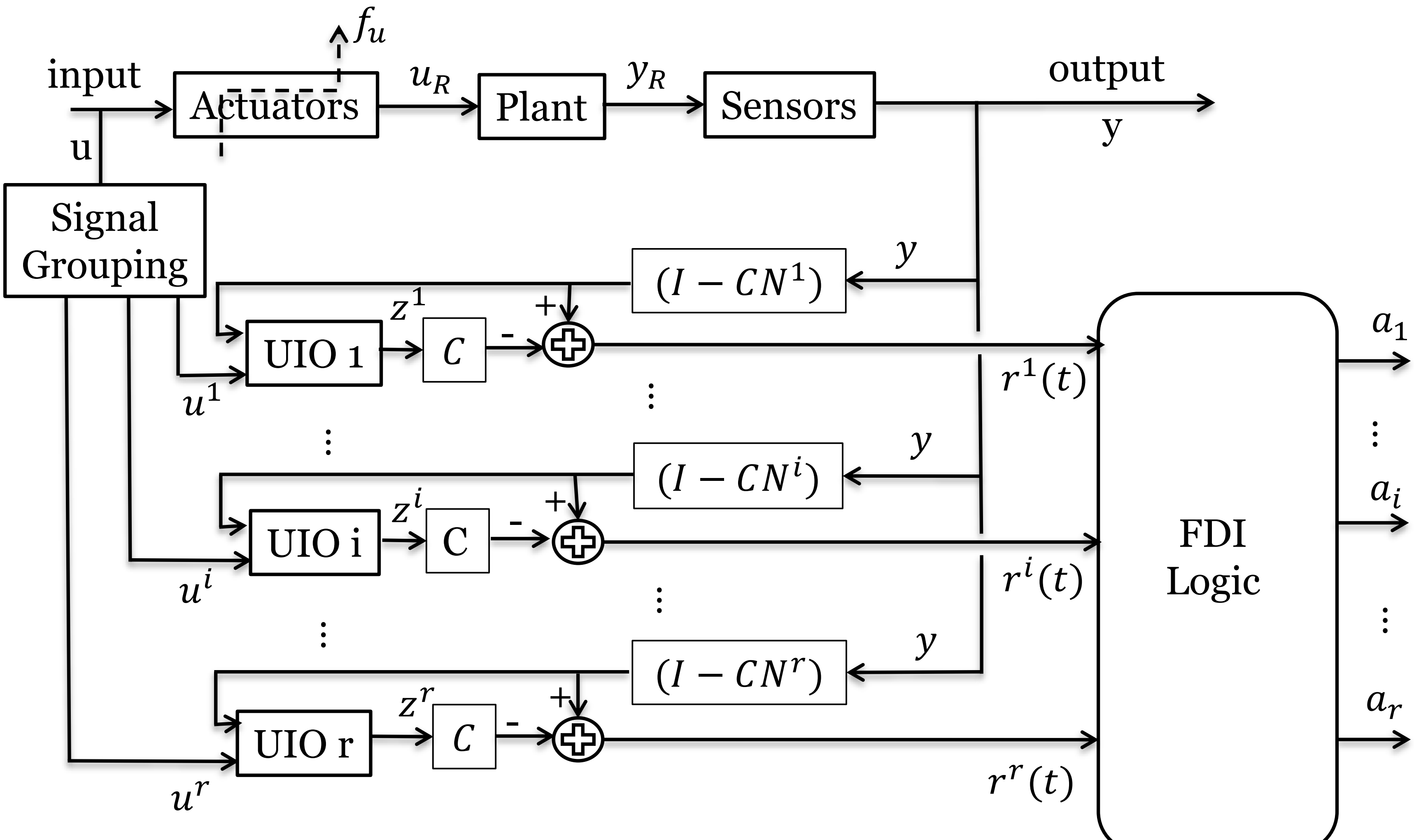
# A robust actuator fault isolation scheme




Where $r^i(t) \in \Re^m$ i=1, 2 ...r

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# Actuator Generalized Observer Scheme

- Analysis of the fault actuator isolability

| | $r^1$ | $r^2$ | … | $r^i$ | … | $r^r$ |
|---|---|---|---|---|---|---|
| $f_{u1}$ | 0 | 1 | 1 | | … | 1 |
| $f_{u2}$ | 1 | 0 | 1 | | … | 1 |
| $\vdots$ | | | $\ddots$ | | | $\vdots$ |
| $f_{ui}$ | 1 | 1 | | 0 | | 1 |
| $\vdots$ | | | | | $\ddots$ | |
| $f_{ur}$ | 1 | 1 | 1 | | | 0 |

⇒ $a_1$=1 if $f_{u1}$ hold true

Table 1: *Generalized Observer Scheme* GOS

In the tables above, a "1" in $i_{th}$ row and $i_{th}$ column denotes that the residual $r_i$ is sensitive to the fault $f_{ui}$, whilst a "0" denotes insensitivity.

- Fault alarm
  - $a_1 = L(\|\bar{r}^1\|) \cap L(\|r^2\|) \cap L(\|r^3\|) \cap \ldots \cap L(\|r^m\|)$

where $L(\|r^i\|)=1$ if $\|r^i\| \geq \varepsilon^i$

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# Robust actuator fault isolation schemes

- Existence condition : In order to satisfy the constraints
  - ✓ $T^i E^i$=0 $c_1$ : decoupled condition
  - ✓ $F^i$=$T^i$A- $K^i$ $C$ stable $c_2$ : detectability condition

the following rank conditions must be verified for each UIO:

- ✓ $c_1$: rank$\left(\begin{bmatrix} CE^i \\ E^i \end{bmatrix}\right)$=rank$\left(CE^i\right)$=rank$(C[E \quad b_i])$=dim(d)+1
- ✓ $c_2$: rank $\left(\begin{bmatrix} pI_n - T^i A \\ C \end{bmatrix}\right)$=n $\forall \ \Re(\lambda(T^i A))\geq 0$

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# A robust sensor fault isolation scheme

## ❑ Remarks

- ✓ The isolation schemes presented in this previous time can only isolate a single fault in either a sensor or an actuator, at the same time.
- ✓ This is based on the fact that the probability for two or more faults to occurs at the same time is very small in a real situation.
- ✓ If simultaneous faults need to be isolated, the fault isolation scheme should be modified based on a regrouping of faults. Each residual will be designed to be sensitive to one group of faults and insensitive to another group of faults. See the following Dedicated Observer Scheme (DOS).

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# Residual Generation Techniques

- ❑ Observer-based approaches
- ❑ Parity (vector) relations
- ❑ Fault detection via parameter estimation

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# Parity relation for fault detection and isolation

- Parity space approach
  - For static system
    - Y=CX
    - without UI
  - For static constraint system
    - AX=0
    - Y=CX
    - without UI
  - For static system without perfect UI decoupled
    - $y_k = Cx_k + \varepsilon_k + Fd_k \Leftrightarrow y_k = Cx_k + \varepsilon_k + F^{-}d^{-}_k + F^{+}d^{+}_k$
    - where $y_k \in \Re^m$, $x_k \in \Re^n$, $\varepsilon_k \in \Re^m$, $d_k \in \Re^p$ are respectively, the measurement vector, state vector, noise vector and fault vector.
    - The fault vector is decomposed of two term, $d^{-}_k \in \Re^{p-1}$ denotes the fault vector to be undetected and $d^{+}_k \in \Re^1$ the fault to be detected
    - Assumption : m > n
  - Dynamic system
    - $x_{k+1} = Ax_k + Bu_k + E^1 d_k + R^1 f_k$
    - $y_k = Cx_k + Du_k + E^2 d_k + R^2 f_k$
    - Where $f_k \in \Re^g$ denotes a fault vector which may contain actuator, component or sensor faults, $d_k \in \Re^q$ denotes the UI (or disturbance) vector.

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# Parity Relation for fault detection

- To begin with this problem, let us consider a general problem of the measurement of an n-dimensional vector using $m$ sensors.
- The algebraic measurement equation is described as:
  - ✓ $y_k = Cx_k + f_k$
  - ✓ Where $y \in \Re^m$ is the measurement vector, $x \in \Re^n$ is the state vector, $f_k$ is the vector of sensor fault and $C$ is an $m \times n$ measurement matrix.
- Two case can be considered
  - ✓ m<n (without redundancy)
  - ✓ m≥n (with redundancy)

# Parity Relation for fault detection

- Case 1 : m<n, $y_k$= $Cx_k$ and *rank(C)* = m, the *direct* redundancy relation does not exist however, we may construct redundancy relations by collecting sensor outputs over a time interval (data window):

  - Y(k-s:k)=$\begin{bmatrix} y_{k-s} \\ y_{k-s+1} \\ \vdots \\ y_k \end{bmatrix}$

- ✓ This is known as "temporal redundancy" or "serial redundancy". An example is done at the end of this section.

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# Parity Relation for fault detection

- Case 2 : m≥n and *rank(C)* = n, this implies that the rows of *C* are linearly dependent, i.e., the outputs of the sensors are related by a static relation.
- Example 1: $Y = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 1 & 1 \\ 2 & 0 \end{pmatrix} X$

  - ✓ Isolate a regular of matrix C, as: $C_1 = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$
  - ✓ Since C1 is invertible, the system becomes $\begin{cases} Y_1 = C_1 X \\ Y_2 = C_2 X \end{cases} \Leftrightarrow \begin{cases} C_1^{-1} Y_1 = X \\ Y_2 = C_2 C_1^{-1} Y_1 \end{cases}$
  - ✓ Which is equivalent to $\begin{pmatrix} -C_2 C_1^{-1} & I \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = 0 \Leftrightarrow \begin{cases} 0.5y_1 + 0.5y_2 - y_3 = 0 \\ 2y_2 - y_4 = 0 \end{cases}$
  - ✓ The last relation gives 2 redundancy relation and a third relation can be obtained $0.5y_1 - y_3 + 0.5y_4 = 0$
  - ✓ Hence, if a fault occur at sensor $y_2$, we obtain $\begin{cases} 0.5y_1 + 0.5y_2 - y_3 \neq 0 \\ 2y_2 - y_4 \neq 0 \\ 0.5y_1 - y_3 + 0.5y_4 = 0 \end{cases}$ and the fault can be detected.

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# Parity Relation for fault detection

- Example 2 : $y_k$=C$x_k + f_k$
- The dimension of $y_k$ is larger than the dimension of $x_k$, i.e. $m > n$ and *rank(C)* = $n$. For such system configurations, the number of measurements is greater than the number of variables x.
- For FDI purposes, the vector $y_k$ can be combined into a set of linearly independent parity equations to generate the parity vector (residual):
  - $r_k = Vy_k = \mathrm{V}(\mathrm{C}x_k + f_k)$
  - ✓ In order to obtain a good characteristic of the residual $r_k$ (zero-valued for the fault-free case and insensitive of the unknown x), the matrix $V$ must satisfy the condition:
    - V×C=0
  - ✓ When this condition holds true, the residual (parity vector) only contains information on the faults
    - $r_k = Vy_k = Vf_k \Leftrightarrow r_k = v_1 f_1(k) + v_2 f_2(\mathrm{k}) + \ldots v_m f_m(k)$
- where $v_i$ is the column of $V$, $f_i$ *is* the $i^{th}$ element of f(k) which denotes the fault in the $i^{th}$ sensor.

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# Parity Relation for fault Isolation

- Example 3 : $y_k = Cx_k + \varepsilon_k + Fd_k \Leftrightarrow y_k = Cx_k + \varepsilon_k + F^- d^-{}_k + F^+ d^+{}_k$
  - ✓ Where the fault vector d is decomposed of two term, $d^-{}_k \in \Re^{p-1}$ denotes the fault vector to be undetected and $d^+{}_k \in \Re^1$ the fault to be detected.
- Ideal solution (exact decoupling)
  - ✓ Rewrite the above system as
    - $y_k = [C \quad F^-]\begin{pmatrix} x_k \\ d^-{}_k \end{pmatrix} + \varepsilon_k + F^+ d^+{}_k$
  - ✓ and find the parity vector P such that
    - $P = \Omega\, y_k = \Omega\, F^+ d^+{}_k + \Omega \varepsilon_k$
    - $\Omega\,[C \quad F^-] = 0$
    - $\Omega\, F^+ \neq 0$
  - ✓ Therefore P is sensitive to $d^+$ and insensitive to x and $d^-$.

Numerical application

$$C = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 0 & 2 \end{pmatrix}, \quad F^- = \begin{pmatrix} 1 & 2 \\ 1 & 2 \\ 0 & 0 \\ 2 & 5 \\ 0 & 1 \end{pmatrix}, \quad F^+ = \begin{pmatrix} 1 \\ 0 \\ 3 \\ 1 \\ 1 \end{pmatrix}$$

- Optimization projection (optimal decoupling)
  - In practice the constraints $\Omega\,[C \quad F^-] = 0$ and $\Omega\, F^+ \neq 0$ are unfeasible.
  - One solution is to find an optimal matrix $\Omega$ such that :

$$\min_{\Omega} \|\Omega\,[C \quad F^-]\| \text{ and } \max_{\Omega} \|\Omega\, F^+\|$$

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# Parity Relation for fault Isolation

## Multi-objective optimization

- One of methods to solve the multi-objective optimization problem is to optimize a new cost function J which accounts for both $J_1 = \min_{\Omega} \|\Omega [C \quad F^-]\|$ and $J_2 = \max_{\Omega} \|\Omega\, F^+\|$ . A solution for minimizing J cannot minimize $J_1$ at the same time as maximizing $J_2$.
- However, it could lead to a reasonable solution for robust residual design.
- A sensible mixture of performance indices is their ratio, i.e.
  - $J = \min_{\Omega} \frac{\|\Omega\, [C \quad F^-]\|^2}{\|\Omega\, F^+\|^2}$
- Hence, the robust residual design is achievable by minimizing J.

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# Parity Relation for fault Isolation

- The problem min max
  - $\min_{\Omega}\|\Omega\,[C \quad F^-]\|$ and $\max_{\Omega}\|\Omega\,F^+\|$

Is transformed as the following optimization problem

- $\min_{\Omega}\frac{\|\Omega\,[C \quad F^-]\|^2}{\|\Omega\,F^+\|^2}$ or $\begin{cases}\Omega C=0\\ \min_{\Omega}\frac{\|\Omega\,F^-\|^2}{\|\Omega\,F^+\|^2}\end{cases}$ or $\begin{cases}\Omega C=0\\ \min_{\Omega}(\|\Omega\,F^-\|^2-k^2\|\Omega\,F^+\|^2)\end{cases}$

- Since C is a full column matrix we can decomposed the matrix C as
  - $\Omega C=0 \Leftrightarrow \left((\Omega_1)^T \quad (\Omega_2)^T\right)\begin{pmatrix}C_1\\C_2\end{pmatrix}=0$
  - *where $C_1$ is a regular matrix*

✓ After some manipulation we obtain

  - $(\Omega_1)^T=-(\Omega_2)^T C_2(C_1)^{-1}$ which implies $\Omega C=0$
  - And $\Omega=\left((\Omega_1)^T \quad (\Omega_2)^T\right)=(\Omega_2)^T\left(-C_2(C_1)^{-1} \quad I\right)$

✓ The constrain optimization problem is reduce to a simple optimization problem

  - $\begin{cases}\Omega C=0\\ \min_{\Omega}\frac{\|\Omega\,F^-\|^2}{\|\Omega\,F^+\|^2}\end{cases} \Leftrightarrow \min_{\Omega_2}\frac{\left\|(\Omega_2)^T(-C_2(C_1)^{-1} \quad I)F^-\right\|^2}{\left\|(\Omega_2)^T(-C_2(C_1)^{-1} \quad I)F^+\right\|^2} \Leftrightarrow \min_{\Omega_2}\frac{(\Omega_2)^T PF^-(F^-)^T P^T\Omega_2}{(\Omega_2)^T PF^+(F^+)^T P^T\Omega_2}$
  - Where $P=\left(-C_2(C_1)^{-1} \quad I\right)$

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# Parity Relation for fault Isolation

- The optimization problem
  - $\min_{\Omega_2} \frac{(\Omega_2)^T PF^-(F^-)^T P^T \Omega_2}{(\Omega_2)^T PF^+(F^+)^T P^T \Omega_2} \Leftrightarrow \min_{\Omega_2} \frac{(\Omega_2)^T A \Omega_2}{(\Omega_2)^T B \Omega_2}$
  - Where A=$PF^-(F^-)^T P^T$ and B= $PF^+(F^+)^T P^T$

✓ is reduced to find the minimum eigenvalue of the pair(A, B) and the corresponding eigenvector.

- Proof
  - $F_1 : \begin{cases} \Re^n \rightarrow \Re \\ X \rightarrow \frac{X^T A X}{X^T B X} = \underline{\gamma}) \end{cases}$
  - $\underline{\gamma}$ is a minimum of $F_1$ for X = $X^*$ iff

✓ $\left.\frac{\delta F_1}{\delta x}\right|_{X=X^*} = 0 \Leftrightarrow \frac{\delta \frac{{X^*}^T A X^*}{{X^*}^T B X^*}}{\delta X^*} = 0 \Leftrightarrow \frac{2AX^*{X^*}^T BX^* - {X^*}^T AX^* 2BX^*}{{X^*}^T B{X^*}^2} = 0 \Leftrightarrow \frac{2}{{X^*}^T BX^*}\left(\text{A} - \underbrace{\frac{{X^*}^T AX^*}{{X^*}^T BX^*}}_{\underline{\gamma}} B\right) X^* = 0$

✓ Which is equivalent to $\left(A - \underline{\gamma} B\right) X^*$=0 where $X^*$ is called the generalized eigenvector associated of the generalized eigenvalue $\underline{\gamma}$

✓ The roots of det($A - \quad B$) denoted by : $\underline{\gamma} < \gamma_j < \ldots < \gamma_i < \bar{\gamma}$ are called the generalized eigenvalues of the matrix $A - \quad B$ and $\underline{\gamma}$ the corresponding minimum eigenvalue.

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# Parity Relation for the dynamic systems

- Consider the discrete-time system with the following description :

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + R_1 f_k \\ y_k = Cx_k + Du_k + R_2 f_k \end{cases}$$

  - ✓ Where $u_k \in \Re^r$ is the known input vector, $y_k \in \Re^m$ is the known output vector and $x_k \in \Re^n$ is the unknown state vector, $f_k \in \Re^g$ denotes a unknown fault vector which may contain actuator, component or sensors faults.
  - ✓ $\{A, B, R_1, R_2, C \text{ and } D\}$ are known real matrices with appropriate dimensions.

- How we obtain the redundancy relations ?

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# Parity Relation for the dynamic systems

- How we obtain the redundancy relations ?
  - ✓ Combining together the above relation from time instant k-s to time instant k yields the following redundant relations:

$$\underbrace{\begin{bmatrix} y_{k-s} \\ y_{k-s+1} \\ \vdots \\ y_k \end{bmatrix}}_{Y_k} - \mathrm{H} \underbrace{\begin{bmatrix} u_{k-s} \\ u_{k-s+1} \\ \vdots \\ u_k \end{bmatrix}}_{U_k} = \mathrm{W} x_{k-s} + \mathrm{M} \underbrace{\begin{bmatrix} f_{k-s} \\ f_{k-s+1} \\ \vdots \\ f_k \end{bmatrix}}_{F_k}$$

  - ✓ where

$$\mathrm{H}=\begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s-1}B & CA^{s-2}B & \cdots & D \end{bmatrix} \in \Re^{(s+1)m\times(s+1)r} \qquad \mathrm{W}=\begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix} \in \Re^{(s+1)m\times n}$$

  - ✓ And the matrix M is constructed by replacing $\{D, B\}$ with $\{R_2, R_1\}$ in the matrix H.
  - ✓ To simplify the notation, the above equation can be written as:
    - $Y_k - HU_k = Wx_{k-s} + MF_k$

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# Parity Relation for the dynamic systems

- From the following data relation:
    - $Y_k - HU_k = Wx_{k-s} + MF_k$
  - ✓ a residual signal can be defined as:
    - $r_k = V[Y_k - HU_k]$
  - ✓ Where V $\in \Re^{p\times(s+1)m}$ and p is the residual vector dimension. The degree s is called the order of the parity relation.
  - ✓ In order to analyze the residual, substituting $Y_k - HU_k = Wx_{k-s} + MF_k$ into $r_k$, we obtain:
    - $r_k = VWx_{k-s} + VMF_k$
  - ✓ In order to make the parity vector useful for FDI, one should make it insensitive to unknown states, i.e.
    - VW=0
  - ✓ To satisfy the fault detectability condition, the matrix V should also satisfy the following condition:
    - VM≠0

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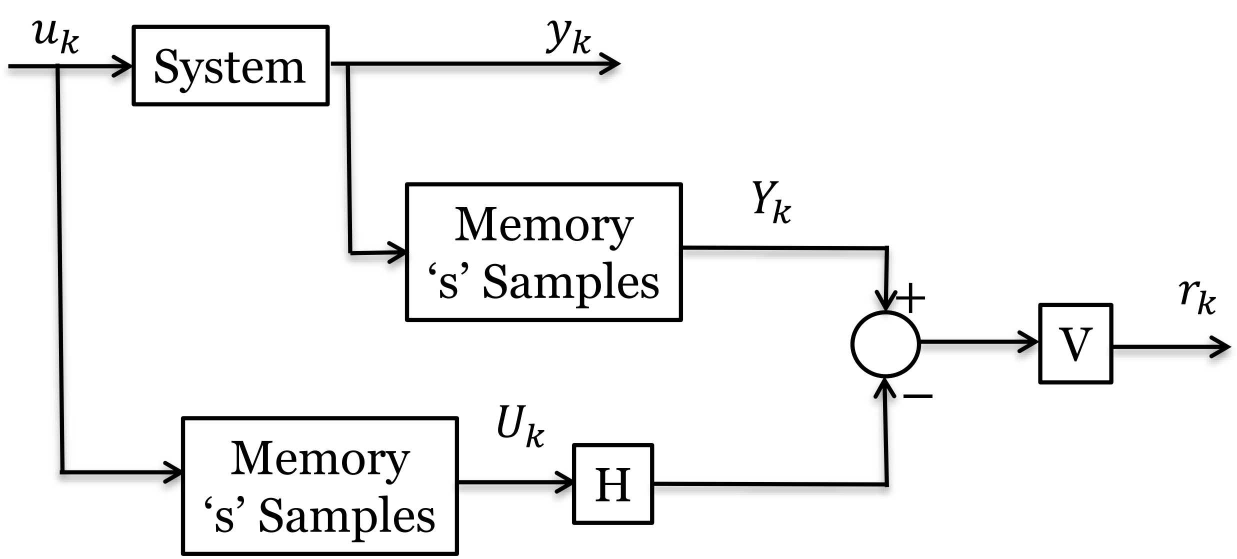
# Parity Relation for the dynamic systems

- The parity relation approach for residual generation of dynamic system is shown in the following figure.




Residual generator via temporal redundancy

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# Parity Relation for the dynamic systems

## FDI using the auto-redundance relation approach

- Consider the following fault sensor model : $\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k + f_k \end{cases}$
- and the decision table

| | $f_1$ | $f_2$ | … | $f_m$ |
|---|---|---|---|---|
| $r_1$ | 1 | 0 | … | 0 |
| $r_2$ | 0 | 1 | … | 0 |
| $\vdots$ | | | | $\vdots$ |
| $r_m$ | 0 | 0 | … | 1 |

- Where
  - $r_{ik} = V_i[Y_{1k} - H_i U_k] = V_i W_i x_{k-s} + V_i M_i F_{ik}$, i=1, 2, …. m

*Constraints :*

$V_i W_i = 0$

$V_i M_i \neq 0$

$$W_i = \begin{bmatrix} C_i \\ C_i A \\ \vdots \\ C_i A^s \end{bmatrix} \qquad H = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C_i B & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_i A^{s-1} B & C_i A^{s-2} B & \cdots & 0 \end{bmatrix} \qquad \underbrace{\begin{bmatrix} f_{ik-s} \\ f_{ik-s+1} \\ \vdots \\ f_{ik} \end{bmatrix}}_{F_{ik}}$$

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# Parity Relation for the dynamic systems

- Consider the discrete-time system with the following description :

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + E_1 d_k + R_1 f_k \\ y_k = Cx_k + Du_k + E_2 d_k + R_2 f_k \end{cases}$$

  - ✓ Where $u_k \in \Re^r$ is the input vector, $y_k \in \Re^m$ is the output vector and $x_k \in \Re^n$ is the state vector, $f_k \in \Re^g$ denotes a fault vector which may contain actuator, component or sensors faults, $d_k \in \Re^q$ is the unknown input (disturbance) vector.
  - ✓ $\{A, B, E_1, E_2, R_1, R_2, C \text{ and } D\}$ are known system model matrices with appropriate dimensions.

- In order to detect and isolate the faults a DOS or GOS scheme could be used.

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# Residual Generation Techniques

- ❑ Observer-based approaches
- ❑ Parity (vector) relations
- ❑ Fault detection via parameter estimation

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# Parameter estimation for fault detection

- The process parameters are not known at all, or they are not known exactly enough. They can be determined with parameter estimation methods
- The basic structure of the model has to be known
- Based on the assumption that the faults are reflected in the physical system parameters
- The parameters of the actual process are estimated on-line using well-known parameter estimations methods
- The results are thus compared with the parameters of the reference model obtained initially under fault-free assumptions
- Any discrepancy (écart) can indicate that a fault may have occurred

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**An approach for modelling the input-output behavior of the monitored system will be recalled and exploited for fault detection**

- SISO model

  $Z=H\underline{\theta}+\varepsilon$

  - $\underline{\theta}$ unknown parameter vector
  - $\varepsilon$ noise of the sensor $\cong N(0,V)$
  - Z sensor
  - H known matrix

Example of a Calorimeter

$$Y=\beta_0+\beta_1 U+\beta_3 U^3$$
$$\Leftrightarrow Z=H\underline{\theta}+\varepsilon$$
$$\Leftrightarrow Y=\begin{bmatrix}1 & U & U^3\end{bmatrix}\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_3\end{bmatrix}+\varepsilon$$

- Criteria

  - $\min_{\theta} J(\ ) = \min_{\theta} \frac{1}{2}\|Z-H\theta\|^2_{V^{-1}} \Leftrightarrow \min_{\theta}\frac{1}{2}(Z-H\theta)^T V^{-1}(Z-H\theta)$
  - $\Leftrightarrow \left.\frac{\partial J(\theta)}{\partial\theta}\right|_{\theta=\hat{\theta}}=0$ and $\frac{\partial J(\theta)}{\partial\theta * \partial\theta^T}>0$

- Solution

  - $\hat{\theta}=(H^T V^{-1} H)^{-1} H^T V^{-1} Z$
  - $\frac{\partial J(\theta)}{\partial\theta * \partial\theta^T}=H^T V^{-1} H>0$

If $V=\sigma^2 I$ then $\hat{\theta}=(H^T H)^{-1} H^T Z=H^+ Z$
It's the Simple LS method

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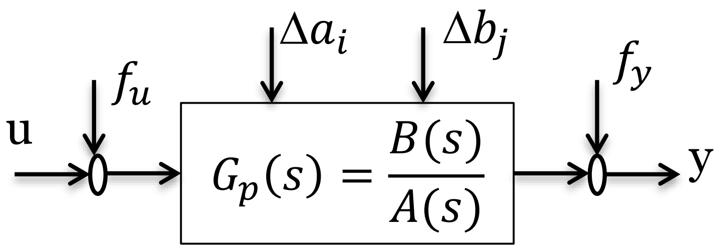
# Parameter Estimation for dynamic system




- Input/output model
  - Measured signals:
    - y(t)=Y(t)-$Y_{00}$; u(t)=U(t)-$U_{00}$ (Linearized around $Y_{00}$, $U_{00}$)
  - Basic equation without faults
    - y(t)+$a_1 y^{(1)}(t) + \cdots + a_n y^{(n)}(t)$=$b_0 u(t)$+$b_1 u^{(1)}(t) + \cdots$+$b_m u^{(m)}(t)$
    - y(t)=$h^T(t)\Theta$
    - $h^T$=$[-y^{(1)}(t) \quad \ldots \quad -y^{(n)}(t) \quad u(t) \quad \ldots \quad u^{(m)}(t)]$
    - $\Theta^T(t)$=$[a_1 \quad \ldots \quad a_n \quad b_0 \quad \ldots \quad b_m]$
  - Additive faults
    - $f_u$ input fault; $f_y$ output fault
  - Multiplicative faults
    - $\Delta a_i, \Delta b_j$ $parameters\ faults$

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# Parameter Estimation for dynamic system

- Spate space model
  - Basic equation

    $\dot{x}(t)$=Ax(t)+bu(t)

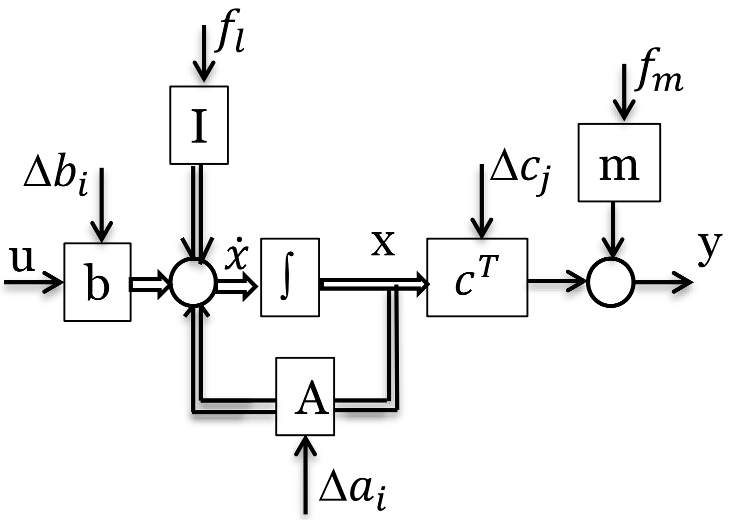
    y(t)= $c^T$x(t)

    $$A=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -a_1 \\ 1 & 0 & -a_2 \\ \vdots & \vdots & \vdots \end{bmatrix}$$

    $b^T=[b_0 \quad b_1 \quad \ldots]$

    $c^T=[0 \quad 0 \quad \ldots 1]$

  - *Additive fault*
    - $f_l$ input or state variable fault
    - $f_m$ output fault
  - Multiplicative fault
    - ΔA, ΔB, Δc parameter faults

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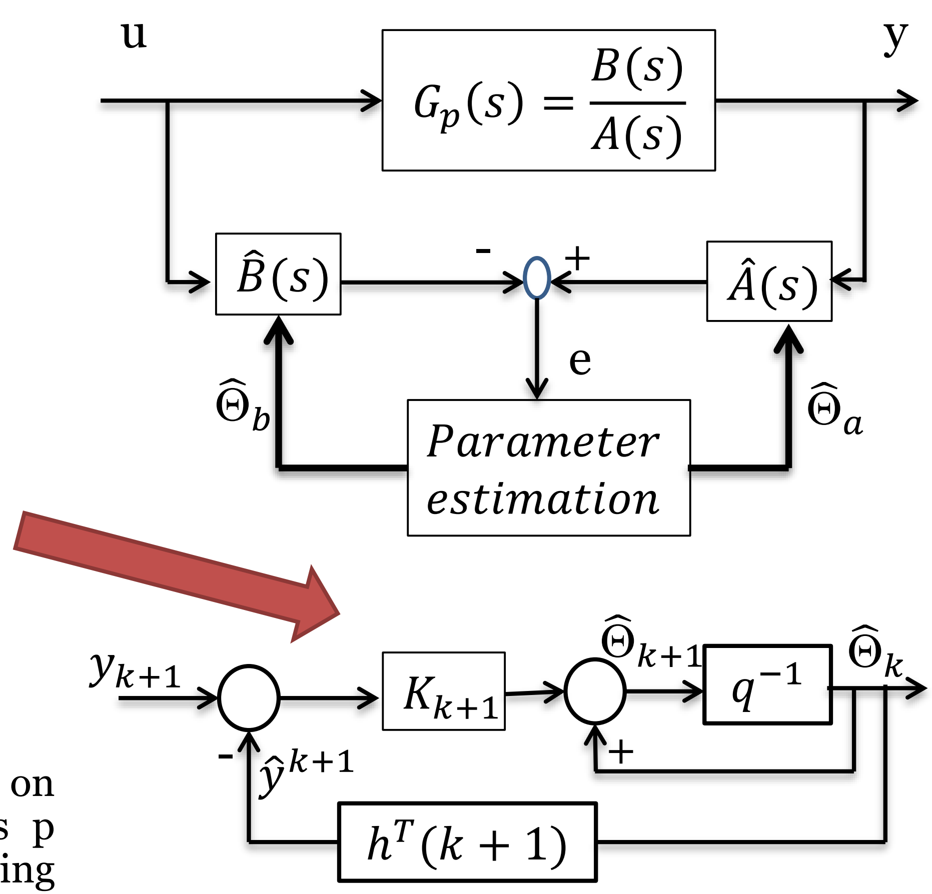
## Minimization of equation error

- Loss function:
  - $V=\sum e^2(k)$
- Method
  - Non-recursive
  - $\widehat{\Theta} = [H^T H]^{-1} H^T y$
  - Recursive
  - $\widehat{\Theta}_{k+1} = \widehat{\Theta}_k + K_{k+1}(y_{k+1} - h^T(k+1)\widehat{\Theta}_k)$
- Symptoms (residuals):
  - Model parameters

    $$\Delta\widehat{\Theta}(j) = \widehat{\Theta}(j) - \Theta_o$$

  - Process parameters $\Theta$=f(p) depend on physically defined process coefficients p (like stiffness “raideur”, damping “amortissement” coefficients, resistance).
  - Process coefficients

    $$\hat{p}=f^{-1}(\widehat{\Theta}) \quad \Delta p(j) = \hat{p}(j)-p_0$$

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- Proof of Non-recursive LS method
  - Consider an ARX model
    - $A(q)y(t) = B(q)u(t) + \varepsilon(t)$

      where

      $A(q) = 1 + a_1 q^{-1} + \cdots + a_{na} q^{-na}$

      $\mathrm{B}(q) = b_1 q^{-1} + \cdots + b_{nb} q^{-nb}$

      $\varepsilon = N(0, \sigma^2)$
  - Basic equation
    - $y(t) = h^T(\mathrm{t})\theta + \varepsilon\,(\mathrm{t})$

      where $h^T$(t)=[-y(t-1) … -y(t-$n_a$) u(t-1) … u(t-$n_b$)] and $\theta^T = [a_1 \ \ldots a_{na} \ b_1 \ \ldots b_{nb}]$
  - Data record

$$
\begin{bmatrix} y(1) \\ \\ y(k) \\ \vdots \\ y(t) \end{bmatrix} = \begin{bmatrix} -y(0) & -y(1) & \cdots & -y(1-na) & u(0) & \cdots & u(1-nb) \\ \\ -y(k-1) & -y(k-2) & \cdots & -y(k-na) & u(k-1) & \cdots & u(k-nb) \\ \\ -y(t-1) & \cdots & & -y(t-na) & u(t-1) & \cdots & u(t-nb) \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_{na} \\ b_1 \\ \\ b_{nb} \end{bmatrix} + \begin{bmatrix} \varepsilon(1) \\ \\ \varepsilon(k) \\ \\ \varepsilon(t) \end{bmatrix}
$$

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# Fault detection with parameter methods for dynamic system

- Define Z=H$\underline{\theta}$+$\underline{\varepsilon}$

where $Z^T(t) = [y(1) \ldots y(t)], \varepsilon^T = [\varepsilon(1) \ldots \varepsilon(t)], \varepsilon = N(0, V = \sigma^2)$

$$H(t) = \begin{bmatrix} -y(0) & -y(1) & \cdots & -y(1-na) & u(0) & \cdots & u(1-nb) \\ -y(k-1) & -y(k-2) & \cdots & -y(k-na) & u(k-1) & \cdots & u(k-nb) \\ -y(t-1) & \cdots & & -y(t-na) & u(t-1) & \cdots & u(t-nb) \end{bmatrix}$$

- Minimize the following criteria
  - $\min_{\theta} J(\theta) = \min_{\theta} \frac{1}{2} \|Z(t) - H(t)\theta(t)\|^2_{V^{-1}}$
- Solution mco
  - $\hat{\theta}(t) = \left[H^T(t)H(t)\right]^{-1} H^T(t)Z(t)$

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- Proof of Recursive LS method
  - ✓ Data record

$$\begin{bmatrix} y(1) \\ \\ y(k) \\ \vdots \\ y(t) \end{bmatrix} = \begin{bmatrix} -y(0) & -y(1) & \cdots & -y(1-na) & u(0) & \cdots & u(1-nb) \\ \\ -y(k-1) & -y(k-2) & \cdots & -y(k-na) & u(k-1) & \cdots & u(k-nb) \\ \\ -y(t-1) & & \cdots & -y(t-na) & u(t-1) & \cdots & u(t-nb) \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_{na} \\ b_1 \\ \\ b_{nb} \end{bmatrix} + \begin{bmatrix} \varepsilon(1) \\ \\ \varepsilon(k) \\ \\ \varepsilon(t) \end{bmatrix}$$

$$y(t+1) \quad \begin{bmatrix} -y(t) & \cdots & -y(t+1-na) & u(t) & \cdots & u(t-nb) \end{bmatrix} \quad \varepsilon(t+1)$$

  - ✓ Which is equivalent to

$$\begin{pmatrix} Z(t) \\ y(t+1) \end{pmatrix} = \begin{bmatrix} H(t) \\ h^T(t+1) \end{bmatrix} \underline{\theta} + \begin{pmatrix} \varepsilon(t) \\ \varepsilon(t+1) \end{pmatrix}$$

  - ✓ From the previous results, we obtain
    - $\hat{\theta}(t+1) = \left[H^T(t+1)H(t+1)\right]^{-1} H^T(t+1)\mathrm{Z(t+1)}$

$$\hat{\theta}(t+1) = \left( \begin{bmatrix} H^T(t) & h(t+1) \end{bmatrix} \begin{bmatrix} H(t) \\ h^T(t+1) \end{bmatrix} \right)^{-1} \begin{bmatrix} H^T(t) & h(t+1) \end{bmatrix} \begin{pmatrix} Z(t) \\ y(t+1) \end{pmatrix}$$

$$\hat{\theta}(t+1) = \left(H^T(t)H(t) + h(t+1)h^T(t+1)\right)^{-1} \left(H^T(t)Z(t) + h(t+1)y(t+1)\right)$$

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# Fault detection with parameter methods for dynamic system

**Matrix inversion Lemma**

$$(A+BCD)^{-1} = A^{-1} - A^{-1}B\left(C^{-1} + DA^{-1}B\right)^{-1}DA^{-1}$$

$$A = H^T(t)H(t), \quad B = h(t+1), \quad C = 1, \quad D = h^T(t+1)$$

- Proof of Recursive LS method
  - ✓ Using the **matrix inversion Lemma**, the relation

$$\hat{\theta}(t+1) = \left(H^T(t)H(t) + h(t+1)h^T(t+1)\right)^{-1}\left(H^T(t)Z(t) + h(t+1)y(t+1)\right)$$

  - ✓ Becomes

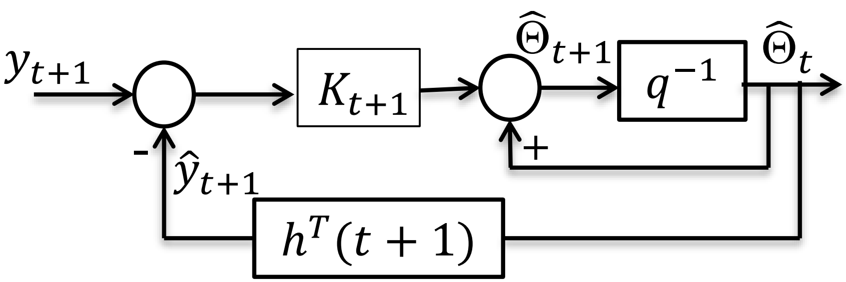
Gain factor

Prediction error

$$\hat{\theta}(t+1) = \hat{\theta}(t) + \frac{\left(H^T(t)H(t)\right)^{-1}h(t+1)}{1 + h^T(t+1)\left(H^T(t)H(t)\right)^{-1}h(t+1)}\left(y(t+1) - h^T(t+1)\hat{\theta}(t)\right)$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + K_{t+1}\left(y(t+1) - \hat{y}(t+1)\right)$$

  - ✓ where




$$K_{t+1} = \frac{P(t)h(t+1)}{1 + h^T(t+1)P(t)h(t+1)}$$

$$P(t) = \left(H^T(t)H(t)\right)^{-1}$$

$$P(t+1) = \left(H^T(t+1)H(t+1)\right)^{-1} = \left(H^T(t)H(t) + h^T(t+1)h(t+1)\right)^{-1} = P(t) - \frac{P(t)h(t+1)h^T(t+1)P(t)}{1 + h^T(t+1)P(t)h(t+1)}$$

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## Details of development

$$\hat{\theta}(t+1)=\left(H^T(t)H(t)+h(t+1)h^T(t+1)\right)^{-1}\left(H^T(t)Z(t)+h(t+1)y(t+1)\right)$$

$$\hat{\theta}(t+1)=\left[\left(H_t^T H_t\right)^{-1}-\left(H_t^T H_t\right)^{-1}h_{t+1}\left(1+h_{t+1}^T\left(H_t^T H_t\right)^{-1}h_{t+1}\right)^{-1}h_{t+1}^T\left(H_t^T H_t\right)^{-1}\right]\left(H_t^T Z(t)+h_{t+1}y(t+1)\right)$$

Since $\hat{\theta}(t)=\left[H^T(t)H(t)\right]^{-1}H^T(t)Z(t)$, the above relation becomes

$$\hat{\theta}(t+1)=\hat{\theta}(t)-\left(H_t^T H_t\right)^{-1}h_{t+1}\left(1+h_{t+1}^T\left(H_t^T H_t\right)^{-1}h_{t+1}\right)^{-1}h_{t+1}^T\hat{\theta}(t)$$

$$+\left[\left(H_t^T H_t\right)^{-1}h_{t+1}-\left(H_t^T H_t\right)^{-1}h_{t+1}\left(1+h_{t+1}^T\left(H_t^T H_t\right)^{-1}h_{t+1}\right)^{-1}h_{t+1}^T\left(H_t^T H_t\right)^{-1}h_{t+1}\right]y(t+1)$$

Which is equivalent to

$$\hat{\theta}(t+1)=\hat{\theta}(t)-\frac{\left(H_t^T H_t\right)^{-1}h_{t+1}}{1+h_{t+1}^T\left(H_t^T H_t\right)^{-1}h_{t+1}}h_{t+1}^T\hat{\theta}(t)+\left[\left(H_t^T H_t\right)^{-1}h_{t+1}-\frac{\left(H_t^T H_t\right)^{-1}h_{t+1}}{\left(1+h_{t+1}^T\left(H_t^T H_t\right)^{-1}h_{t+1}\right)}h_{t+1}^T\left(H_t^T H_t\right)^{-1}h_{t+1}\right]y(t+1)$$

$$\hat{\theta}(t+1)=\hat{\theta}(t)-\frac{\left(H_t^T H_t\right)^{-1}h_{t+1}}{1+h_{t+1}^T\left(H_t^T H_t\right)^{-1}h_{t+1}}h_{t+1}^T\hat{\theta}(t)+\left(H_t^T H_t\right)^{-1}h_{t+1}\left[1-\frac{h_{t+1}^T\left(H_t^T H_t\right)^{-1}h_{t+1}}{\left(1+h_{t+1}^T\left(H_t^T H_t\right)^{-1}h_{t+1}\right)}\right]y(t+1)$$

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## Details of development

$$\hat{\theta}(t+1)=\hat{\theta}(t)-\frac{\left(H_t^T H_t\right)^{-1} h_{t+1}}{1+h_{t+1}^T\left(H_t^T H_t\right)^{-1} h_{t+1}} h_{t+1}^T \hat{\theta}(t)+\left(H_t^T H_t\right)^{-1} h_{t+1}\left[1-\frac{h_{t+1}^T\left(H_t^T H_t\right)^{-1} h_{t+1}}{\left(1+h_{t+1}^T\left(H_t^T H_t\right)^{-1} h_{t+1}\right)}\right] y(t+1)$$

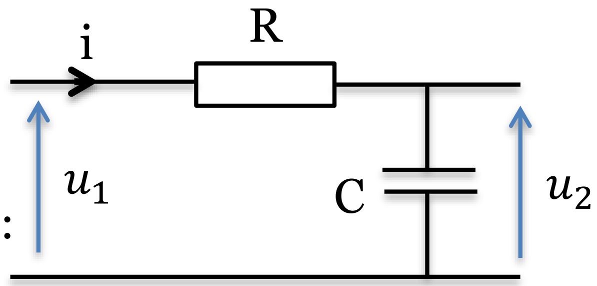
Put it under the same denominator

$$\hat{\theta}(t+1)=\hat{\theta}(t)-\frac{\left(H_t^T H_t\right)^{-1} h_{t+1}}{1+h_{t+1}^T\left(H_t^T H_t\right)^{-1} h_{t+1}} h_{t+1}^T \hat{\theta}(t)+\left(H_t^T H_t\right)^{-1} h_{t+1}\left[\frac{1+h_{t+1}^T\left(H_t^T H_t\right)^{-1} h_{t+1}-h_{t+1}^T\left(H_t^T H_t\right)^{-1} h_{t+1}}{\left(1+h_{t+1}^T\left(H_t^T H_t\right)^{-1} h_{t+1}\right)}\right] y(t+1)$$

Simplified the numerator, the RLS is obtained

$$\hat{\theta}(t+1)=\hat{\theta}(t)+\frac{\left(H^T(t) H(t)\right)^{-1} h(t+1)}{1+h^T(t+1)\left(H^T(t) H(t)\right)^{-1} h(t+1)}\left(y(t+1)-h^T(t+1)\hat{\theta}(t)\right)$$

# Application of the LS method




- Consider the following circuit system
  - ✓ Assign $b_0 = 1, b_1 = C \; and \; a_1 = RC$
- In case 1, we consider $u_1$ and $u_2$ as the output measurements :
  - ✓ $b_0 \, u_1(t) = u_2(t) + a_1 \frac{du_2(t)}{dt}$
  - ✓ Using the LS method we obtain an estimate of $\theta^T = [b_0 \;\; a_1]$.
  - ✓ The process coefficient $\hat{p} = f^{-1}(\widehat{\Theta})$ is not available
- In case 2, we consider $u_1$ and $i$ as the output measurements :
  - ✓ $b_1 \frac{du_1(t)}{dt} = a_1 \frac{di(t)}{dt} + i(t)$
  - ✓ Using the LS method we obtain an estimate of $\theta^T = [b_1 \;\; a_1]$.
  - ✓ The process coefficient $\hat{p} = f^{-1}(\widehat{\Theta})$ is available : $\hat{p}^T = [R \;\; C]$

- Exercise :
  - ✓ Describes the LS procedure
  - ✓ Gives the parameter residual

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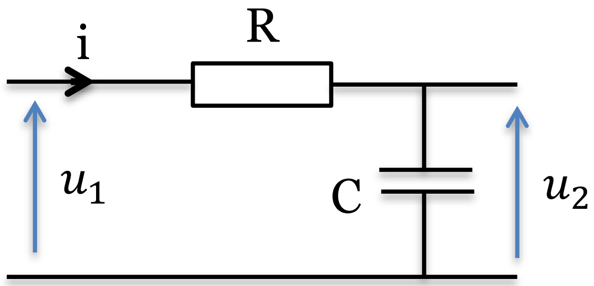
# Application of the LS method




## ❑ Case 2 :

- ✓ $C\,\frac{du_1(t)}{dt} = RC\,\frac{di(t)}{dt} + i(t)$
  - Using an implicit Euler approximation
- ✓ $C\,\frac{u1^{n+1}-u1^{n}}{\Delta t} - RC\frac{i^{n+1}-i^{n}}{\Delta t} = i^{n+1}$
- ✓ $i^{n+1} = \left(-\frac{i^{n+1}-i^{n}}{\Delta t} \quad \frac{u1^{n+1}-u1^{n}}{\Delta t}\right)\binom{RC}{C}$

Numeric modelisation :
(Mathematical Lecturer of UTC)

$$[M]\{\dot{T}\} + [K]\{T\} = \{F\}$$

$$\{\dot{T}\} = \frac{\partial\{T\}}{\partial t} = \frac{\{T\}^{n+1} - \{T\}^{n}}{\Delta t} + \Delta t(\ldots)$$

1. Schéma EXPLICITE :

$$[M]\frac{\{T\}^{n+1} - \{T\}^{n}}{\Delta t} + [K]\{T\}^{n} = \{F\}^{n}$$

2. Schéma IMPLICITE :

$$[M]\frac{\{T\}^{n+1} - \{T\}^{n}}{\Delta t} + [K]\{T\}^{n+1} = \{F\}^{n+1}$$

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